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MARCO RAO
Department of Fusion and Technology for Nuclear Safety and Security
Division of Technology Applications for Security, Health and Heritage
Development of Particle Accelerators and Medical Applications Laboratory
Frascati Research Centre, Rome

G. LUTERO
ISTAT, Quarterly National Accounts

THE UNPREDICTABILITY AND BLACK SWANS

The impact over forecasting and seasonal adjustment in economic short-term modeling

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MARCO RAO
Department of Fusion and Technology for
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Medical Applications Laboratory
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G. LUTERO
STAT, Quarterly National Accounts

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The impact over forecasting and seasonal adjustment in economic short-term modeling

Giancarlo Lutero, Marco Rao

Abstract

Official statistics and short-term data are an increasingly strategic factor in decision support for a country's energy and economic policy. The official short-term published data need models and methods that are timely, but above all, reliable in order to reduce estimation errors, especially to counter those events characterized by systemic uncertainty and by so-called black swans. This paper illustrates how unpredictability can have a significant impact on short-term data, in particular seasonally adjusted data, and on economic statistics, using the aggregation problem and the various types of uncertainty as a framework, and the impact of unpredictability on the quality of forecasts and published data.

Key words: *Unpredictability, Black Swans, Aggregation Method, Forecast Error, Official Statistics, Seasonal adjustment, Outliers.*

Riassunto

Le statistiche ufficiali ed i dati congiunturali sono un fattore sempre più strategico nel supporto decisionale alla politica energetica ed economica di un paese. I dati congiunturali ufficiali pubblicati hanno bisogno di modelli e di metodi che siano tempestivi ma soprattutto affidabili, al fine di ridurre gli errori di stima, specialmente per contrastare quegli eventi caratterizzati da incertezza sistemica e dai cosiddetti black swans. In questo lavoro viene illustrato come l'incertezza possa avere un notevole impatto sui dati congiunturali, in particolare quelli destagionalizzati, e sulle statistiche economiche, utilizzando come framework il problema dell'aggregazione e le varie tipologie d'incertezza, e l'impatto dell'imprevedibilità sulla qualità delle previsioni e dei dati pubblicati.

Parole chiave: Incertezza, Cigni Neri, Metodi di Aggregazione, Errore di Previsione, Statistiche Ufficiali, Destagionalizzazione, Osservazioni Anomale.

Summary

INTRODUCTION 7
2. DATA AND METHODS 9
  2.1 THE AGGREGATION PROBLEM: A SUMMARY INTRODUCTION 9
  2.2 THE AGGREGATION OF LONGITUDINAL-SPATIAL DATA 10
  2.3 THE AGGREGATION IN FORECASTING 14
3 UNPREDICTABILITY AND THE BLACK SWAN PROBLEM 17
4 EMPIRICAL EVIDENCE 22
  4.1 EXPENDITURE BY NON-RESIDENT IN ITALY 30
5 CONCLUSIONS AND DISCUSSIONS 33
BIBLIOGRAPHY 34

Introduction

Official statistics released by national office of statistics are an increasingly strategic factor in decision for policy-making in country's energy and economics, especially in consideration to the relationship between economic growth, climate change [1][2], and socio-economic costs of the disastrous events [3][4]. Especially short-term data need models and methods that are timely reliable in order to reduce estimation and prediction errors, in particular to counter those events characterized by systemic uncertainty, and by that particular event identified by the concept of B*lack Swans*. In the comparison between basic data and macroeconomic data the aggregation is a crucial, and often underrated, step in estimation process [5][6][7][8]. The access to high quality information is fundamental in decision-making, so the aggregation procedure is crucial, since it could result in policy making misleading. The topic of aggregation (section 2.1) is going to be exposed under several points of view; according to their features are defined three typologies of data aggregation: longitudinal or spatial aggregation, temporal aggregation, contemporaneous aggregation. In section 2.2 the issue of aggregation of longitudinal data is discussed in greater detail: for several scholars, to work with aggregated data produces several positive effects as simplicity and parsimony of analysis, faster statistical elaboration, completeness and reliability of statistical sources, cost savings for new microdata surveys. On the other hand, working with disaggregated models and micro-data, has a lot of positive feedback for the users: informative completeness, better estimates, models specification heterogeneity, a potential better prediction.

In section 2.3, aside from the identification of the most efficient data to work with, we will discuss another decisive issue for the choice of a statistical model: the forecasting capability in different temporal and environmental contexts [9][10][11]. A key question in forecasting is whether the point forecast of an aggregate (direct method) improves upon those derived from a combination of forecasts with a specific, parametric, or not, aggregation rules (indirect approach). Statistical theory did not give a clear indication of the optimal method. In this context of uncertainty, forecast combination technique [12] is a recommended solution, due to risk diversification principle, especially when decision makers have an aversion respect to dimension of forecast errors, or against the persistence of direction in prediction error.

After a review on the aggregation problem and an in-depth on problems caused by predictive failure of stochastic models, the axiomatic definition of unpredictability [13][14] is outlined in section 3. *Unpredictability* is defined as intrinsic stochastic variation in a known distribution, where available information does not alter the unconditional distribution. Three types of uncertainty in stochastic processes will be exposed and discussed: intrinsic, instance and extrinsic unpredictability. One of the most characteristic phenomena of recent times is a particular kind of error in statistical induction, the so-called *black swans*, will be introduced [15][16]. They are featured by independent highly discrepant draws from fat-tailed, or heavy-tailed distributions, representing rare events, where there is a non-negligible probability of large outcomes, in terms of forecast error, and potentially social and economic large costs.

The process of estimating short-term data [20] can be heavily influenced by further data transformation operations such as the most important and tricky, the seasonal adjustment [21][22][23][24], and others like temporal disaggregation or deflation of current values, to express the data in real terms. An estimation with official quarterly data is presented in section 4: two time series that have been impacted, in terms of compromission of stability and forecasting capacity of statistical models, by an event that we can undoubtedly classify as a black swan, particularly in the step where the data are adjusted to remove the seasonal component. It will be shown how the statistical offices had to intervene to try to contain the distortion effect introduced in the stochastic models by the Covid-19 event, through the use of a pattern of dummy variables or intervention regressors. The last section 5 summarizes the issues discussed here.

2. Data and methods

This section begins with a discussion of the theoretical foundations of the treatment of uncertainty in the statistical literature, with particular attention to the problem of aggregation. The concept of unpredictability, as mainly discussed by Hendry, introduces the concept, fundamental in this work, of the black swan.

2.1 The aggregation problem: a summary introduction

Aggregation is one of more crucial, and often underrated, step in statistical data estimation. The timely access to high quality data is fundamental in decision-making, so the aggregation procedure is crucial since it could result in policy making misleading. The topic may be investigated under several points of view: according to specific features, and with a certain degree of abstraction, are available three typologies of aggregation

1. longitudinal or spatial aggregation, namely aggregation across geographical space, or aggregation of physical and institutional units or productive sectors;
2. temporal aggregation, that involves the passage from higher frequency data (i.e., quarterly) to lower one (typically, annual data);
3. contemporaneous aggregation, in which aggregation is made across variables (i.e., the construction of composite index as the Human Development Index) or between outcomes of several models to forecasting purposes (i.e., the combining or pooling forecasting[1]).

[1] See [**12**] for an introduction to forecast combinations methodologies.

To work with highly aggregated data, produce some positive effects for analysis: the most significant are that in some cases, is simplicity and parsimony. Moreover, statistical elaboration is faster, that is a fundamental point to produce timely statistics. Aggregated data address the problem of incomplete or unreliable statistical sources and its impact on model specification, because often aggregation introduces systematic error. Finally, collection and elaboration of new micro-data from new surveys are quite expensive and pose further constraints since it is required to evaluate their informative contribution. On the other hand, working with disaggregated models and micro-data lead to an informative completeness, better estimates[2], a model specification heterogeneity (micro-data availability allows to consider various model specifications across micro units, improving the fitness of the model), and finally better predictions, thanks to higher micro-data accessibility and better and differentiated micro specifications.

2.2 The aggregation of longitudinal-spatial data

In the representation of economic facts, problems originated by aggregation is one of most controversial and debated in econometrics and statistics, starting from seminal contributions of Theil [5], Malinvaud [6], Grunfeld and Griliches [7], Zellner [8] and Leontief [17]. Theil introduced for the first time the concept of *aggregation bias* that is defined as deviation of aggregated linear model parameters from average of corresponding micro parameters, in a systematic way. Formally, issue of aggregation bias concerns possibility that, relating to true β parameter value, asymptotic bias may be greater in the aggregated estimator rather than in disaggregated one, that is:

1. $\left|\text{plim}\left(\hat{\beta}_d - \beta\right)\right| < \left|\text{plim}\left(\hat{\beta}_a - \beta\right)\right|$

[2] Greater number of available observations involves more degrees of freedom in data treatment, and, subsequently, more efficient estimates, more powerful statistical tests.

Here we consider only univariate linear models and finite sample. Following Theil [5], let us consider disaggregated model relative to a single micro-unit i (i.e., country, industry, household) at time t:

2. $y_{it} = x_{it}\beta_i + u_{it} \qquad i = 1, \dots, n \quad t = 1, \dots, T$

where y_{it} is dependent variable relative to unit i at time t, $x_{1it} = (x_{it}, \dots, x_{kit})$ is a vector of k exogenous variables, β_i is vector of coefficient for the i-th section and u_{it} is a stochastic error with usual hypothesis (mean equal zero and variance σ_i). Assuming linear additive model, the aggregation equation corresponds to

3. $y_{at} = \sum_{i=1}^{n} y_{it} = x_{at}\beta_a + u_{at}$

To identify the statistical correspondence between micro-equations and macro-equation perturbations, Theil adopts a set of auxiliary relations, computing projections of micro-model predictor variables on the corresponding of aggregated ones:

4. $x'_{it} = Z_n\delta_i + \varepsilon_{it}$

where matrix $Z_n = \mathrm{diag}(x_{iat}, x_{2at} \dots, x_{kat})$ and ε_{it} are auxiliary regression shocks. Finally, substituting the equation (4) in structural micro equation (2) and summing across units, the estimated aggregate perturbations become the sum of two effects, respectively the aggregation bias and sampling disturbances:

5. $\hat{u}_{at} = \sum_{i=1}^{n} u_{it} = \sum_{i=1}^{n}\left(\varepsilon'_{it}\hat{\beta}_i + \hat{u}_{it}\right)$

Hence, variance of aggregate error is the following:

6. $\hat{\sigma}_a^2 = n^{-1}\left(\sum_{i=1}^{T} \varepsilon'_{it}\hat{\beta}_i\right)^2 + \sum_{i=1}^{T} \hat{\sigma}_i^2 + \sum_{j\neq i} \sigma_{ij}$

From equation 6 is inferable that variance of aggregation bias can be computed as a weighted mean, whose weights are represented by estimated squared micro coefficients. When first sum vanishes is obtained the so-called consistent or perfect aggregation (outputs, in terms of goodness of fit, are approximately the same, in relation to target), that may occur in the following cases[3]:

- micro homogeneity[4]: all micro parameters are equal to aggregated one, so they are constant over the longitudinal dimension;
- compositional stability: we talk about joint probability distribution of exogenous variables, and it occurs when the predictor composition across units do not change over time;
- symmetric distribution: predictors are white noises;

Theil's analysis assumes strong hypothesis on correct specification of both models, aggregated and disaggregated. Basing to this last assumption, Grunfeld and Griliches [7] proposed that grouping data may in some cases lead to an improvement in terms of unbiased estimates and efficiency (aggregation gain), evaluating models using a goodness of fit criterion based on $\overline{R}^2$, or considering efficient prediction. These authors conclude that disaggregated models are likely misspecified, i.e., because the set of micro equations, often the structural equation, do not capture the dynamic behavior of macroeconomic variables, or because micro-data could present measurement errors[5]: in these cases, data aggregation is not "necessarily bad".

Particularly good article of Pesaran, Pierse and Kumar [10] resumes state-of-the-art on static linear aggregation and proposes a more general misspecification test, comparing the differences between average of estimated micro coefficients and aggregated estimate. Null hypothesis of perfect aggregation is the following:

7. $\xi = \sum_{i=1}^{n} X_i \beta_i - X_a b = 0$

If $\xi > 0$ this statistic test might be taken as measure of misspecification in micro equations.

[3] Unfortunately, these conditions turn out to be rather stringent: the reported cases are almost matter of pure mathematics and have scarce economic significance.
[4] This is hypothesis formulated by Zellner for perfect aggregation test, introduced in context of SURE estimator, see **[8]**.
[5] See Aigner and Goldfeld **[18]**.

Stoker [28] stressed that is necessary to define a complete aggregation structure between macro aggregate and micro functions to detect a behavioral interpretation of the former, that is to say there should be a one-to-one correspondence between macro function and micro functions. Completeness is a statistical feature, which allows to extend and to incorporate analysis of nonlinear micro-behavior and to deepen distribution of predictors across units[6].

Lippi and Forni [19] expand investigation to dynamic models' specification, and showed that adopting an ARMAX framework for micro equations as follows:

8. $\phi(L)y_{it} = \gamma(L)x_{it} + \theta(L)\varepsilon_{it}$

with usual stochastic assumptions and no common polynomial factors, it is possible to improve study of individual heterogeneity, providing for relations between dynamic structures of several micro equations, represented by polynomials in lag operator $\phi(L), \gamma(L), \theta(L)$. Besides, they stress that dynamic form is altered in the passage from micro to macro equations (dynamic effects of aggregation[7]) and, even though aggregated model might possess much profound dynamic specification, well-specified disaggregated models give in general better outcomes.

The problems of the distortions introduced by aggregation become very complicated when the data undergoes various operations, which correspond to functional transformations, such as the removal of seasonality (seasonal adjustment), the temporal disaggregation or the final reconciliation of the data itself: these intervention operations on the data emphasize once again whether it is better to adopt a direct approach in statistical estimation, or an indirect approach where all these operations are carried out on the data disaggregated (by geographical area, by production sector, etc.)

[6] For example, density classes of exponential family are complete in relation to aggregation.

[7] These dynamization effects may arise from errors in set of available variables, from temporal aggregation, from omission of relevant variables and finally from adoption of linear framework to interpret nonlinear relationships.

One of the most significant problem faced in short-term data domain is what would happen if some proxy-indicators were not available for the most recent period, generally the most interesting one for users. One or two months of the current quarter could be not available at the time of publication of data: in this case, using forecasting methods is necessary to reconstruct missing information and proceed with the subsequent steps of the estimation process. A key question in forecasting is whether the point forecast of an aggregate (direct method) improves upon those derived from a combination of forecasts with a specific, parametric, or not, aggregation rules (indirect approach). Regarding to theoretical considerations and basing on empirical results available in the large econometric literature, a clear preference for direct or indirect forecasting approach does not emerge. Forecasts' performances depend on many issues like the type of statistical models used, the features of time series, the forecasting horizon, and many other factors.

For instance, David Hendry in many of his works[8] establishes that the sources of forecast or prediction errors, may arise for many reasons, like for example model specification or variables identification to include in models, stochastic model functional form (essentially linear model versus nonlinear), model selection criteria for dynamic models, general estimation uncertainty, data measurement errors or, last but not least, presence of structural breaks over forecast horizon. Therefore, in macroeconomic forecasting practice is necessary to exclude from the analysis of outcomes the great impact of many events which considerably influence results like aggregation bias, presence of structural breaks or a clear trending, or the use of too many predictors, the impact of seasonal adjustment procedure and data revision.

[8] For example, see Hendry **[13][14]**.

A point forecast $\hat{y}_{t+h|t}$, with forecast horizon h, is usually defined as the value which minimizes the mean squared error, given the available information set Ω at time T:

25. $\hat{y}_{t+h|t} = E\,(y_{t+h}|\Omega_t)$

forecast or prediction error is defined as follows

26. $e_{t+h|t} = y_{t+h} - \hat{y}_{t+h|t}$

Among several available forecast accuracy measures, we may consider the most popular like:

the Mean Absolute Error

27. $MAE = E(|e_{t+h}|)$

the Mean Squared Error

28. $MSE = E\left(e_{t+h}^2\right)$

or the Mean Error, especially useful to study possible persistent asymmetries in the empirical distribution of forecast error

29. $ME = E(e_{t+h})$

and finally, the Root Mean Squared Error

30. $RMSE = \left[E\left(e_{t+h}^2\right)\right]^{1/2}$

It is crucial to pay attention to the distribution of the forecast error, to select the most efficient stochastic model, or to examine alternative measures like i.e., the median values or, for example, also to set an empirical distribution in which a percentage of tails values are discarded.

A recommended solution, especially in the context of official statistics and univariate estimates domain, is the implementation of a forecast combination technique. Several arguments support the adoption of combination forecasts methods: Timmermann [12] lists several of them:

- portfolio diversification due to incomplete information about target variable;

- presence of structural breaks in sample;
- misspecification bias of unknown source and form in individual forecasting model;
- different approach of forecasters;
- pure technical reason like computational capability.

Using forecast combinations for data prediction, especially in official macroeconomic estimation, is a good practice in several contexts. For example, when there is a great uncertainty about the best forecasting model, or to achieve a most efficient methodology; Forecast combinations is a good choice if there is an aversion with respect to large size of forecast errors, or finally when decision makers have a great aversion against the persistence of sign in prediction error.

In very general terms, combination of forecasts includes the following essential steps:

1. selection of models to implement in the process;
2. single model estimate (specification, diagnostics, computational problem);
3. selection of loss function;
4. forecasting summary (aggregation law).

The most delicate and complicated step is certainly the choice of a rule for aggregating estimates. There are several aggregation rules, the most known are the following

- arithmetic mean (which means adopting the hypothesis of equal weights for all available estimates);
- a weighted averages;
- use of factorial methods or cluster;
- adoption of auxiliary regressions;
- the parametric estimates of dynamic weights (Time-varying approach)
- trimming procedure (cutting the tails or the worst models)

The naïve solution, as arithmetic mean and static approach seems to be a particularly good, and practical, solution in presence of high uncertainty. Nonetheless, how is uncertainty defined? A description and an axiomatic definition will be the subject of the next section.

3 Unpredictability and the Black Swan problem

Forecasting ability has always played a fundamental role in the evaluation of a statistical or mathematical model and in applied sciences, but for a long time now, this property has become predominant, if not exclusive, and the only thing object of evaluation. Two problems arise: the first concerns the dogmatic imposition, without any critical sense, of the axiomatic method, of mathematics and its criteria, for both natural and social sciences. This operation could be hazardous, because in this way there is a risk that those epistemological values, rightly considered essential in the exact sciences (i.e., formal coherence, formal logical rigor, coherence, and completeness of the hypotheses, etc.), can degenerate into methodological prejudices. The second concerns the specific differences that occur between forecasts in the environmental or socio-economic field and those valid in the context of exact sciences, which are significantly connected to the active intervention of human subjectivity in the context of a specific system, subject to prediction.

Why may forecasting fail? The sources of forecast errors in the context of temporal data (the most known macroeconomic statistics are of this type) can be very diversified, as reported in sec. 2.1.2: from the problems of identification and specification of the variables to the functional form (linear vs nonlinear) selected for the models. From a general uncertainty in the events, to measurement errors in the input data or to the presence of structural breaks in the forecast horizon. All this can be concentrated in an inductive fallacy dilemma that can be caused using wrong or insufficient tools as David Hendry rightly points out. The most sensational case of undue generalization, produced by an error in induction, is the one that in recent years has been identified with the metaphor of the Black Swan. This symbolism dates to British philosophy and to John Stuart Mill and was taken up again last century by Karl Popper, but today it is much discussed, even outside academic circles, thanks to work of the mathematician and financial consultant Nicholas Nassim Taleb [15][16]. With Black Swan we designate an almost impossible event in a statistical sense, which is therefore placed on the so-called tails of the observed statistical distributions, with a probability measure almost close to null value (but not equal to zero, here is the difficulty), Whose fulfillment has a disruptive effect on the environment or statistical population analyzed. Consequences are the material damages caused by the failure to evaluate these potential events, and the downsizing of explanatory power attributed to many stochastic models and complex methodologies, especially those adopted in the economic and financial field, which do not precisely consider the impact of black swans.

However, to be more rigorous, there are not only black swans in the framework of statistical uncertainty. As reported by econometrician Robert Engle, Hendry has carefully described the predictive failure of time-series data. Following the suggestions of Hendry, we will try to give an axiomatic definition of uncertainty, or more precisely unpredictability. Hendry defines unpredictability as "intrinsic stochastic variation in a known distribution, where conditioning on available information does not alter the outcome from the unconditional distribution"[9]. He recognizes three classes of uncertainty in stochastic processes

- intrinsic unpredictability
- instance unpredictability
- extrinsic unpredictability

Let us define intrinsic unpredictability as an n-dimensional vector random variable ε that is unpredictable respect to information set τ_{t-1}, when the conditional distribution

[9] Hendry **[14]**.

$D_{\epsilon_t}(\epsilon_t | \tau_{t-1})$ equals the unconditional distribution $D_{\epsilon_t}(\epsilon_t)$ so that:

31. $D_{\epsilon_t}(\epsilon_t | \tau_{t-1}) = D_{\epsilon_t}(\epsilon_t) \quad \forall t \in \tau$

This intrinsic unpredictability also involves the same indifference in mean and variance:

32. $E_t(\epsilon_t | \tau_{t-1}) = E_t(\epsilon_t)$ and $V_t(\epsilon_t | \tau_{t-1}) = V_t(\epsilon_t)$

Denoting the expectations formed at time t as $E_t\,[\cdot]$, and variance is denoted $V_t\,[\cdot]$, for each point in t, and assuming the relevant moments exist. This is the most abstract definition of uncertainty, where the information set is so non-informative, or very incomplete, that it is impossible to obtain an improvement in the performances of model, conditioning it to the available information: so, a variable that is intrinsically unpredictable, cannot be modeled, or forecast better, than its unconditional distribution. On the contrary, instead a variable that is not inherently unpredictable, may still be unpredictable in another ways, because of two additional aspects of uncertainty that we proceed to define with the other two cases.

The instance unpredictability is characterized by independent, highly discrepant, draws from fat-tailed or heavy-tailed, virtually asymmetric, distributions that represent rare events[10], and there is a non-negligible probability of large outcomes, in terms of projection error, and potentially social and economic large costs: this case study in several context is informally called black swan. In this occurrence, the timing and the magnitude of forecast error are then unpredictable, even when the type of the distribution is notorious, and it is constant over time. The transition from statistical Gaussian models to more sophisticated models involve the consideration of heavy tailed distributions, the likely jumps or shift in the series, and the influence of time-varying volatility.

The next reason is instance unpredictability, which arises when the draw of $\epsilon_{t+1|t}$ induces outcomes y_{t+1} that are far from the forecast by $\hat{y}_{t+1|t}$ in the metric of Ω_ϵ, so that:

[10] For instance, the power-law distributions that involves the Pareto distribution, the lognormal, the Laplace or double-exponential distribution.

33. $\hat{\epsilon}_{t+1|t} = y_{t+1} - \hat{y}_{t+1|t}$

The black swan is featured by its rarity, its isolation-singularity over time: it is an even rarer event that groups of contiguous (flocks) black swans can occur. Another fundamental feature of black swan is the duration of its impact on the stochastic process: this influence could be transitory and have reversible effects on the series, which after a few periods, could return to its ordinary path. Nevertheless, there is also another option to consider: the process could undergo an irreversible, definitive shift, affecting the average level of the series, or the moments above the second (skewness, kurtosis). The real problem when black swans come true is not the lack of sophistication of statistical models, but the underestimation of event risk by those who make important economic and environmental decisions.

The Extrinsic unpredictability arises from an unanticipated shift in known distribution, especially the classic Gaussian, at unanticipated times. The vector variable (ϵ_t) is an extrinsically unpredictable process over a period T if there are intrinsically unpredictable shifts in its distribution:

34. $D_{\epsilon_{t+1}}(\cdot) \neq D_{\epsilon_t}(\cdot) \quad \text{for some } t \in T$

This kind of uncertainty, as mentioned, produces a permanent, irreversible shift of the stochastic process and its realizations: this is what is defined structural break in econometric-statistical discipline. Unlike black swans, in this type of uncertainty, there are no sample extractions from types of stochastic variables, but the unpredictability also manifests itself with a simple, a priori known, Gaussian distribution. There are many similarities and links between instance and extrinsic unpredictability. For example, an event that has instance unpredictability in the differenced variables involves extrinsic unpredictability in the levels, and vice versa. Moreover, the unanticipated shifts of the distribution itself at unanticipated times so inherently also involves instance unpredictability, of which location shifts are usually the most pernicious. As mentioned, black swans are empirically represented by the occurrence of a totally unpredictable outliers in time series. The wider the impact of this outlier, in relation to its span, the more the two types of uncertainty overlap.

The definition differences within several types of unpredictability involve dissimilar decision about environmental analysis, energy policy, and forecasting approach in selection of statistical modeling. In presence of intrinsic unpredictability, the lack of information makes conditioning unnecessary, therefore the unconditional distribution and modeling is the best choice for the decision makers. With instance unpredictability (black swans), the distributional form could be known, then there would be some type of predictability, but the outcomes are very discrepant in terms of projection error and social and economic costs, at an unexpected time. Finally, the extrinsic unpredictability regards the unanticipated shift of known statistical distribution that is persistent in placement and historically irreversible in outcomes.

Moving on to the operational phase, there may be information sets that could be useful for predicting the outliers or the shift in the data. In presence of a shift in the series a good ex post solution is the inserting of a deterministic indicator variable or a more complex dummy variables (transitory change dummy, a level shift dummy, or a dummy variable for the so-called ramp effect), so there is a potentially difference between the step of model selection and the forecasting moment. When the causes of shift are well known, the jump of distribution can be modeled by inserting variables of specific shape: the insertion of new variables does not completely solve the forecast problems if they are unable to predict new outliers that affect the series with permanent effects.

The pandemic event of 2020 provides us with a social phenomenon to verify what has been exposed in the previous sections. The event is probably still ongoing and therefore we cannot assess its social, health and economic costs: we know that they exist but they are not yet fully quantifiable. However, assessments can be made on the impact the pandemic has had on the models we use to estimate, for example, official statistics. As mentioned, it has entailed significant costs, the extent of which is still unknown, but above all it has temporarily compromised the stability and forecasting capacity of statistical models, in any context. Adopting the definitions of unpredictability formalized in the previous section, we could define *Covid-19* as something that lies between *instance unpredictability* and *extrinsic unpredictability* because the impact on the dynamics of the short-term series is substantially similar. The biggest unknown is whether irreversible effects will occur in some contexts (health sector, some economic sectors linked to tourism), and consequently on economic and energetic data, with a real paradigm shift in the social and economic data, or if there it will be a phase, the nature of which is still unknown, of returning to a dynamic and systematic path. In this situation of extreme uncertainty, a control strategy on anomalous observations (outliers)[11] on official time series data was necessary, especially for the period first quarter 2020-first quarter 2022.

In particular how managing the effects of the pandemic at the end of the time series in the monthly and quarterly short-term economic indicators? The turbulent phase started in March 2020 and for some indicators is still in progress, (for example tourism sector), and it was faced in an overall coherent manner between the various indicators and survey domains. The choice fell on the use of the impulse dummies approach (*Additive Outliers*) considered as a

[11] See Chen-Liu [27] for an introduction to this issue.

valid alternative to the adoption of the projected component factors and easy to implement in production. Very briefly, an *outlier* is defined as follows

$$\alpha_j \lambda_j(B) I_t(t_j)$$

where α_j is the unknown parameter, $I_t(t_j)$ is the Indicator function

$$I_t(t_j) = \begin{cases} 1, & t_j \in E \\ 0, & t_j \notin E \end{cases}$$

and $\lambda_j(B)$ may assume the following conformation

$$\lambda_j(B) = \begin{cases} 1 & & \textit{Additive outlier AO} \\ \dfrac{1}{1-\delta B} & 0 < \delta < 1 & \textit{Temporary change TC} \\ \dfrac{1}{1-B} & & \textit{Level shift LS} \end{cases}$$

More specifically, from an operational point of view, the choice was that of using consecutive AOs according with their statistical and economic significance: on the occasion of every new data release, the automatic research of an Additive Outlier on the last observation was carried out.

The process of estimating official macroeconomic series is based on the use of stochastic processes, generally generalized linear models of the Arima type. The Arima parametric model is an example of a *black box* model, related to a signal deriving from mathematical considerations (approximation theorems and spectral factorization) on the representation of stationary random signals, but not linked in any way to the physical nature of the signal considered. The most complex and tricky operation in this process is the so-called *seasonal* *adjustment of data*[12]: it is necessary to emphasize that all institutional, but also not official, infra-annual data releases and published by national office of statistics are seasonally adjusted data, expressed at real chain-linked values . Following a signal-extraction paradigm[13] based on Arima models, the economic series are likely characterized by configurations (*unobservable latent components*) with typical characteristics as

- *trend*, or the long-term evolutionary dynamics;
- the *cyclical fluctuations* (3-5-10 years);
- the *seasonal movements*;
- the *irregular component*, which represents the unsystematic dynamics of the series

All the short term data released by the national statistical institutes are estimated in a way that the seasonal component is identified and removed. Why are macroeconomic time series broken down? To carry out economic analysis and discover the structural evolutionary dynamics of the production system, or to facilitate inferential activity, separating short-term dynamics from medium-long term ones, but the most important reason is to find those typically economic factors (cycle-trends)[14], which may be influenced by the international economic situation, by the state of technological application of scientific progress, by economic and energetic policy choices, and finally to estimate the impact of those typically seasonal factors, therefore linked to calendar or climatic and energetic factors. The model-based method *Tramo-Seats*[15] is a mixed parametric procedure for seasonal adjustment, officially used by Italian National Statistics Office (ISTAT), which compared to procedures based on ad-hoc filters has the following advantages

[12] See Kaiser and Maravall [21], Burman [22], Gómez V. and Maravall [23], Planas [24] for an introduction of seasonal adjustment concept and methods.
[13] Burman [22].
[14] See Burns A.F. and Mitchell [20], Kaiser and Maravall [21] for the analysis of business cycle.
[15] See Burman [22], Gómez V. and Maravall [23] as review on Tramo-Seats.

- all assumptions are explicit;
- the relevance of these assumptions can be verified;
- the structural approach of non-observable components allows for statistical inference;
- identification of anomalous observations;
- filter decomposition with optimality features

Tramo-Seats procedure and other methods like *X13-Arima-Seats* or the purely ad-hoc filter methods (*X11*), are today implemented in a platform, developed by Eurostat and the Central Bank of Belgium in Java language, called *JDemetra+*[16]: the estimates here reported are made with the latter in GUI version[17]. Both series, related to tourism expenditures, are a good example of how an unexpected event can create significant identification problems on many statistical models and in particular on its forecasting capabilities, which in this context are essential not only for economic planning, but they are an essential element for estimating the seasonal component and the other factors, especially the last 8-12 observations: the reason is that the final estimate of all components, especially the seasonal one and the seasonally adjusted, is based on the application of the *Wiener-Kolmogorov filter*[18], an optimal, convergent, centered, bilateral and symmetrical filter, which in order to be applied on the latest (and the first) observations requires many out-of-sample predictions, as already mentioned at least 2-3 years. The shift of two series, easily visible during the two-year period 2020-21 (figure 1 and 4), and the large fluctuactions, has a strong impact on the diagnostics of the models, on the forecasts, and above all, on the revision error of these predictions.

[16] See Grudkowska [25], Ladiray D. and Palate [26]: JD+ is developed in Java, the portability property allows its use on different platforms and to create connections with many other high-level programming languages, such as the mathematical and statistical ones. The R platform uses Java classes already compiled by the JVM, which are the core of the JD + client, with significant advantages for subsequent use on a very diverse mix of platforms, operating systems and languages.
[17] See the link https://github.com/jdemetra/jdemetra-app for the references about the software JDemetra+.
[18] See Kaiser and Maravall [21], Planas [24].

In this context it is performed a radical revision of strategy used in the 2020-21 two-year period, characterized in both series by the imposition of a succession of 8 Additive Outliers (eight dummy regressors) over the period, adopting the rule of automatic identification of the model (AMI strategy), with contextual check for anomalous observations on the whole sample. The results confirm the difficulty of finding a solution to best represent the impact of Covid-19 in terms of identifying the model, the diagnostics, the likelihood of forecasts, the set of outliers identified in the two-year period 2020-2021. The results of procedures are still good: series are very similar to each other, parameters and external regressors (dummies) are all largely statistically significant, in both series the shifts are managed with a set of four differentiated dummy variables, discovered by Chen-Liu procedure[19]. In figure 2 is reported the graph about the spectral decomposition of Hotel and restaurants expenditures' original linearized series: trend-cycle and seasonal factors are easily and clearly estimated, by means of WK filter, while the purely random component is almost absent.

Seasonal Adjustment is a methodology that exploits all the information available on a historical series (time and frequency domain): one or more new observations are available at each period. This affects the estimation process causing a revision of the data that is controlled by the user. The estimation error in the preliminary estimator can be broken down into two types of errors: the *final estimation error* (e_{it}), which is an intrinsic error in the stochastic model identified by the procedure, and the *revision error* ($r_{it|t+k}$), caused by the structure of the WK filter. and the quality of the forecast

$$\tau_{it|t+k} = e_{it} + r_{it|t+k}$$

The revision error decreases with the passage of time, until it vanishes for a high k value (say 3, 4 years), when the sequence of preliminary estimators tends towards the final estimator.

[19] Chen C. and Liu [27].

The difficulties introduced by the pandemic event are very well represented by the graph that compares the final estimator (red line) and the average of the preliminary estimators (blue points) of strangers expenditure seasonally adjusted data (see figure 5): a large *forecast error*, and consequently a *revision error*, is clearly visible between the second quarter of 2020 and the second quarter of 2021. The same revision effect can also be identified to a greater extent in the long-term trend component, because of the presence of a large number of anomalous observations in the final part of the sample, specifically the level shift that affect the average value of the treated series.

The effects of the pandemic have not yet worn off, we are not yet able to define when this extraordinary period will come out, and despite this, an intervention strategy must be defined for the next few years. Probably after 4-5 years we could operate with *ad-hoc intervention variables* to model the impact of this black swan. The developer of Tramo-Seats procedure, the spanish statistician Augustin Maravall associated to Victor Gomez, affirms that in occasion of preliminary knowledge of a phenomenon, it would always be desirable to use an ad-hoc regressor instead to leave to algorythm included in Tramo-Seats the choice of how to deal with extraordinary events give rise to anomalous observations: this already puts us in one uncomfortable situation in relation to the treatment of the pandemic period.

Although they may have the same structural feature, it should be stressed that there are conceptual differences between *outliers* and *intervention variables*: briefly the intervention variables

- can have the same structure as an outlier, or a mixture of them, but they differ in the reasons for their use;
- the key feature is that there is a priori information on the event to be corrected;

- it can be entered even if it is not statistically significant, but it is important by a point of view of the representation of the socio-economic phenomenon

The use of outliers, on the other hand, is justified by the fact that

- correct non-linearities that prevent having model residues RegArima nid (Gaussian and independent);
- remove spurious effects on Autocorrelation function and distortion on model parameters and on forecasts;
- they are not identifiable a priori

In the coming years, the producers of official statistics will have to make delicate decisions on how to intervene in a situation that is increasingly uncertain.

Hotel and restaurants expenditures

Figure 1

Hotel and restaurants expenditures, raw and seasonally adjusted data at current prices, values in millions of Euro, sample 1996q1-2021q4

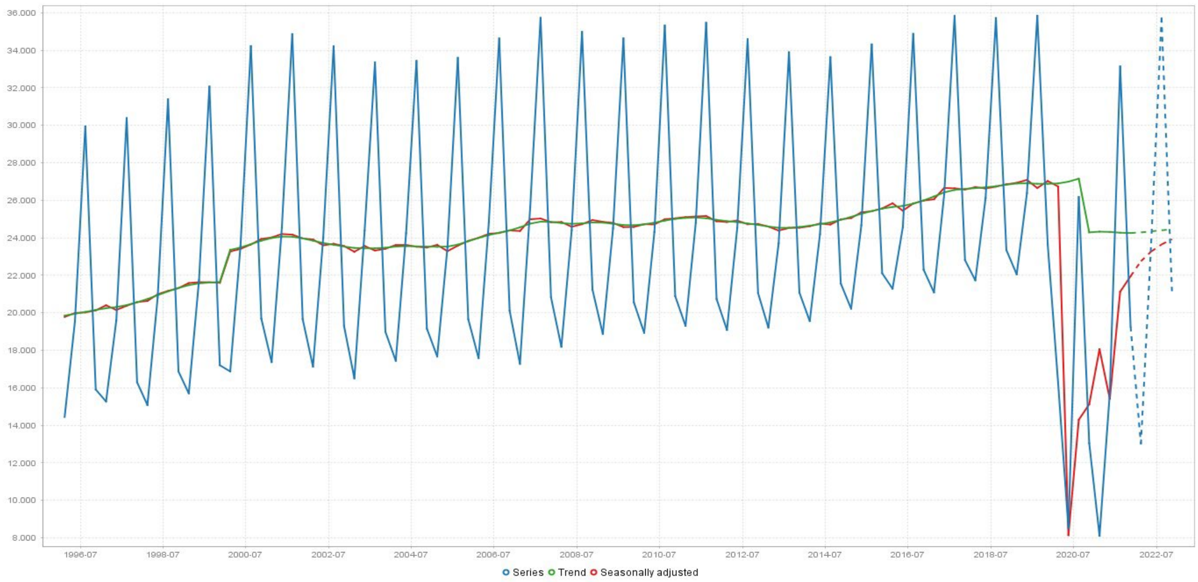


Figure 2
Hotel and restaurants expenditures, decomposition of linearized series spectra

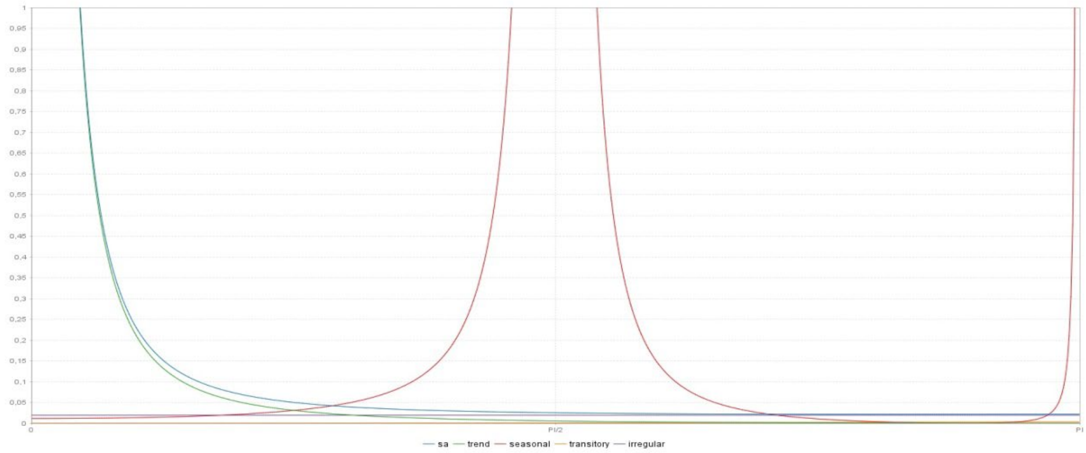
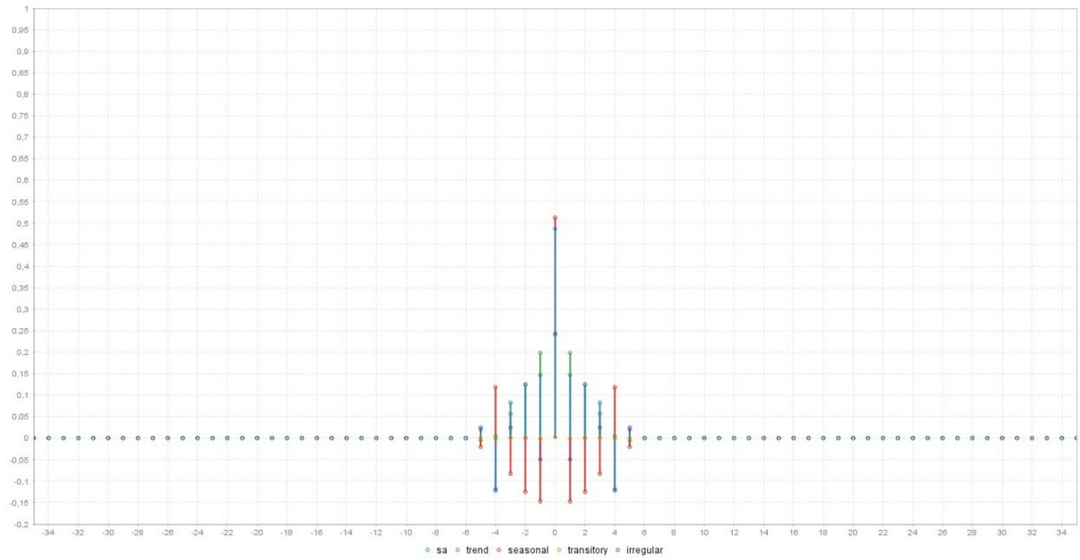


Figure 3
Hotel and restaurants expenditures, Wiener-Kolmogorov filter weights for decomposition



Summary

Estimation span: [I-1996 - IV-2021]
104 observations
No trading days effects
No easter effect
5 detected outliers

Final model

Likelihood statistics

Number of effective observations = 100
Number of estimated parameters = 8

Loglikelihood = -778.4136720231448
Standard error of the regression (ML estimate) = 581.1253510860496

AIC = 1572.8273440462897
AICC = 1574.4097616287072
BIC (corrected for length) = 13.052294881338169

Scores at the solution

-0,003744

Arima model

[(1,0,0)(0,1,0)]

	Coefficients	**T-Stat**	**P[|T| > t]**
Phi(1)	-0,2128	-2,10	0,0384

Regression model

Mean

	Coefficient	**T-Stat**	**P[|T| > t]**
mu	222,0129	2,79	0,0064

Outliers

	Coefficients	**T-Stat**	**P[|T| > t]**
TC (II-2020)	-18676,1641	-36,17	0,0000
SO (I-2020)	-5525,9594	-9,66	0,0000
LS (IV-2020)	-2966,1342	-7,67	0,0000
AO (II-2021)	-4416,0174	-6,03	0,0000
LS (I-2000)	1645,8159	4,57	0,0000

4.1 Expenditure by non-resident in Italy

Figure 4

Expenditure by non-resident in Italy, raw and seasonally adjusted data at current prices, values in millions of Euro, sample 1996q1-2021q4

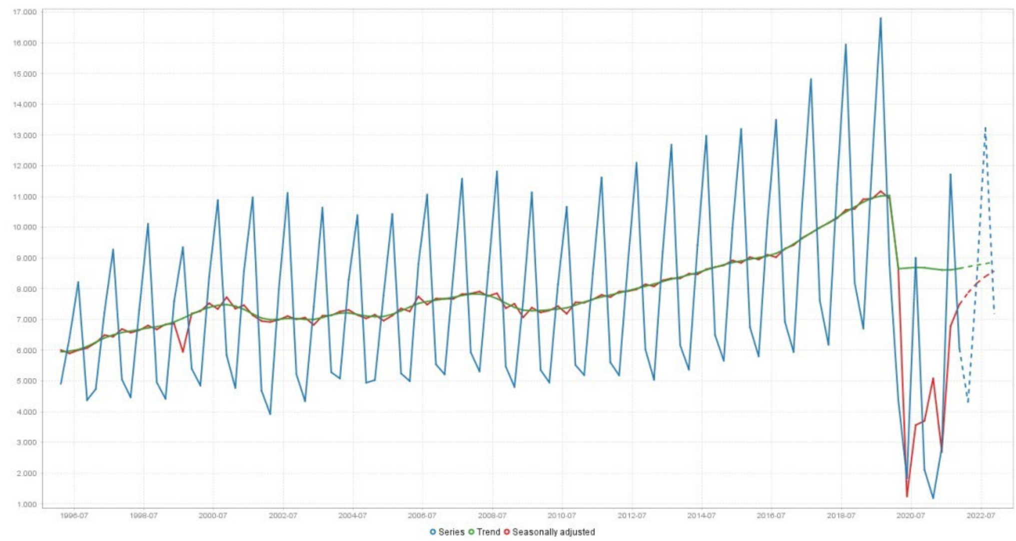
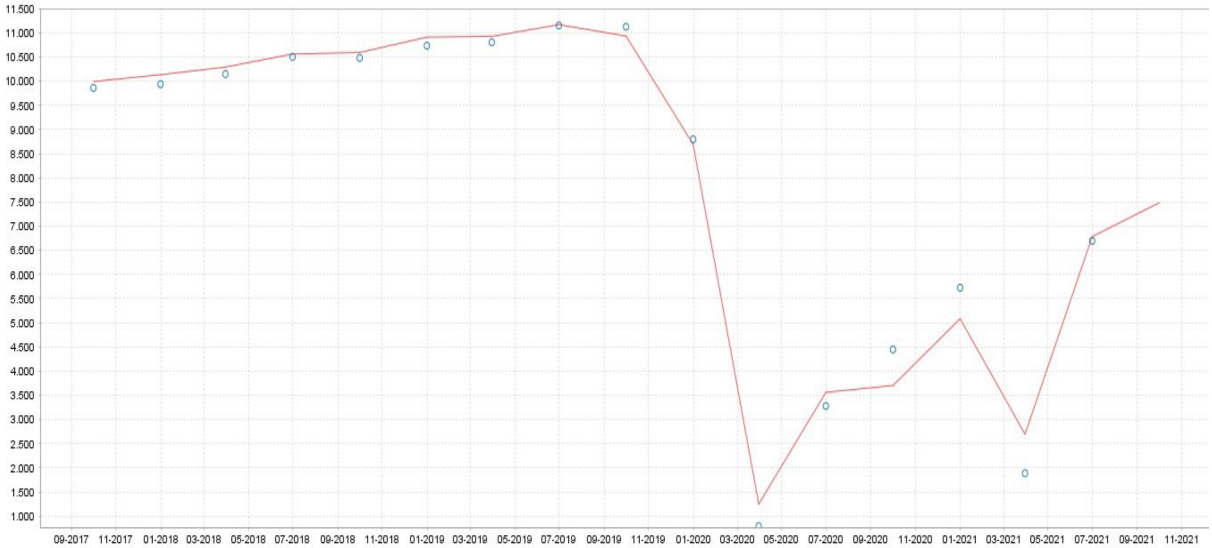


Figure 5

Expenditure by non-resident in Italy, preliminary and final estimator, revision of seasonally adjusted data



Summary

Estimation span: [I-1996 - IV-2021]
104 observations
No trading days effects
No easter effect
5 detected outliers

Final model

Likelihood statistics

Number of effective observations = 100
Number of estimated parameters = 9

Loglikelihood = -740.8901834494189
Standard error of the regression (ML estimate) = 397.1922542582361
AIC = 1499.7803668988379
AICC = 1501.7803668988379
BIC (corrected for length) = 12.337254477119929

Scores at the solution

0,00396 0,006714

Arima model

[(1,0,1)(0,1,0)]

	Coefficients	T-Stat	P[|T| > t]
Phi(1)	0,3019	2,52	0,0135
Theta(1)	0,8984	16,65	0,0000

Correlation of the estimates

	Phi(1)	Theta(1)
Phi(1)	1,0000	0,5492
Theta(1)	0,5492	1,0000

Regression model

Mean

	Coefficient	T-Stat	P[|T| > t]
mu	198,2936	3,23	0,0017

Outliers

	Coefficients	T-Stat	P[|T| > t]
TC (II-2020)	-7391,5755	-20,00	0,0000
LS (I-2020)	-2400,3969	-7,82	0,0000
AO (II-2021)	-3454,9069	-9,20	0,0000
TC (IV-2020)	-1410,9942	-4,14	0,0001
AO (III-1999)	-1092,9745	-4,08	0,0001

5 Conclusions and Discussions

The analysis proposed in this work comes from considerations on the role of the unpredictability in the statistics production and on the use of such a statistic in decision support for economic and energy policymaking. Several problems have been presented that affect the estimates of macroeconomic data: the problem of aggregation in the quality of the data, the impact that the same aggregation has on the prediction capabilities of the stochastic processes defined on that data, the multifaceted concept of uncertainty. To complete the analysis, estimates regarding the Italian tourism and public businesses sector were presented, in order to show how in the two-year period 2020-2021 an event classifiable as a black swan has upset the diagnostics of most of the models used in these contexts: as a case study it was shown how covid-19 has enormously complicated the phase of estimating seasonally adjusted data, introducing biases in the published data, which will be subject to major revisions in the coming years. It is not yet possible to formulate the strategy to be adopted in the next few years because other uncertain events, other black swans, are unfolding and other observations (information) are needed to obtain more stable estimates. It will probably be necessary to operate in the models with a diversified pattern of regressors, that is, a set of dummy variables or ad-hoc specific intervention variables. In the future years, the producers of official statistics will have to make difficult decisions on how to manage a situation that is increasingly uncertain, due to the multiplication of black swans, in a world where the dominant paradigm is becoming that of perpetual social, energetic and economic emergency.

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