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Indirect nonlinear interaction between toroidal Alfvén eigenmode and ion temperature gradient mode mediated by zonal structures

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Abstract

The indirect nonlinear interactions between toroidal Alfvén eigenmode (TAE) and ion temperature gradient mode (ITG) are investigated utilizing nonlinear gyrokinetic theory and ballooning mode decomposition. More specifically, the local nonlinear ITG mode equation is derived adopting the fluid-ion approximation, with the contributions of zonal field structure and phase space zonal structure (ZS) beat-driven by finite amplitude TAE accounted for on the same footing. The obtained nonlinear ITG mode equation is solved both analytically and numerically, and it is found that the ZS beat-driven by TAE only has a weak destabilizing effect on ITG, contrary to usual speculations and existing numerical results.

Keywords: toroidal Alfvén eigenmode, ion temperature gradient mode, zonal structures, gyrokinetic theory, burning plasma

(Some figures may appear in colour only in the online journal)

1. Introduction

Drift wave (DW) turbulence [1] and shear Alfvén wave (SAW) [2–7] are two major categories of collective oscillations contributing to anomalous cross-field transport in magnetized

plasmas. DWs are typically micro-scale electrostatic oscillations driven by free energy associated with thermal plasma density and/or temperature nonuniformities and are widely accepted as candidates for inducing bulk plasma transport. On the other hand, SAW instabilities or, more precisely, Alfvén eigenmodes (AEs) due to equilibrium magnetic geometry excited by EPs, are expected to play a crucial role in EP transport. DW and SAW instabilities are characterized by different spatiotemporal scales, being driven to instability by different free energy sources, and dominating the transport in different energy regimes, and thus were typically studied separately in previous publications. However, it is expected that there are complex cross-scale couplings among DWs and

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SAWs in burning plasmas, due to the mediation by EPs [7, 8]. Specifically, on the one hand, EPs drive meso-scale SAW instabilities that can provide nonlinear feedback to both the macroscopic plasma profiles and microscopic fluctuations. On the other hand, EPs can linearly and nonlinearly (via SAWs) excite zonal structures (ZSs) [9–11], thus acting as generators of nonlinear equilibrium [12, 13]. There is now a rising interest in the mutual effects between them, due to the recent observed thermal plasma confinement improvement in the presence of EPs [14, 15], e.g. experiments and simulations have indicated that the presence of EPs can significantly suppress ion temperature gradient (ITG) turbulence, thereby reducing ion thermal stiffness and enhancing thermal ion confinement [15]. Previous linear gyrokinetic simulations suggested that the presence of EPs might trigger internal transport barriers [16] formation, which could decrease core turbulence levels and improve bulk plasma confinement.

There are several channels for the linear stabilization of micro-turbulence by EPs via, dilution of destabilizing bulk ions [17], EP-pressure induced Shafranov shift stabilization [18], resonant stabilization [19, 20], electromagnetic stabilization [16, 21], and so on. The linear stabilization effects are now relatively well understood, and interested readers may refer to a recent review for a more complete picture [22].

Moreover, numerical studies have revealed a nonlinear mechanism for fast-ion-enhanced EM-stabilization of ITG [23], which could be much more stabilizing than linear ones [20], so these nonlinear coupling mechanisms have drawn significant research interest [24]. A potential physical interpretation is that the presence of EPs provides marginally linearly stable EM-modes, i.e. toroidal Alfvén eigenmodes (TAEs) [25] that transfer energy nonlinearly to ZS [26, 27], and the increase in ZS levels stabilizes the ion-scale turbulence [24, 28]. However, the underlying physical processes of this problem remain to be explored.

More recently, the direct nonlinear interactions between TAE and electron drift wave (eDW) have been proposed and analyzed. It is found that direct nonlinear scattering by ambient eDWs may significantly reduce or even completely suppress TAE stability, due to radiative damping of the nonlinearly generated high- n KAW quasi-modes [29]. On the other hand, for typical reactor parameters and fluctuation intensity, the ‘inverse’ process, i.e. the direct nonlinear scatterings of eDW by ambient TAEs, have negligible net effects on the eDW stability [30], which differs from the results of previous simulations. To understand the bulk plasma confinement in the presence of EPs, as a natural step forward, in this work, we investigate the in-direct nonlinear interaction between TAE and DWs mediated by ZS [31].

ZS, including zonal flows (ZFs), zonal currents (ZCs), and more generally, phase space zonal structures (PSZS) [12, 32], are known to play a significant role in regulating micro-scale DW, or drift Alfvén waves (DAWs), type instabilities [31, 33, 34]. The regulation is realized through the nonlinear generation of ZS by DWs/DAWs, which then scatters DWs/DAWs

into the linearly stable domain with shorter radial wavelengths. The ZS excitation can be achieved via the spontaneous excitation of the radial envelope modulation of DWs/DAWs [31, 33], as well as the beat-driven process of DW/DAW self-coupling. To simplify the analysis while focusing on the main physics picture of nonlinearly generated ZS on DW stability, throughout this paper, the beat-driven process by TAE [35, 36] is taken into account, without including the spontaneous excitation process. Analytical and numerical results have already indicated that the zero frequency radial electric field is capable of significantly stabilizing ITG through the modification of the ‘potential well’ depth [37]. Here, using ITG as the paradigmatic model and following the analysis of [37], we will investigate the indirect nonlinear interaction between TAE and DW, mediated by ZS. Technically, this is achieved by deriving the expression of the TAE beat-driven ZS, and taking it as a ‘nonlinear’ equilibrium into the ITG eigenmode equation. Thereafter, the local dispersion relation and mode structure of the ITG under the influence of ZS is derived in the ballooning space. Here, ‘local’ indicates that the ITG eigenmode equation is dealt with by following the path of the magnetic field lines. Correspondingly, the physics associated with the radial envelope modulation is systematically left out of consideration in our work.

The structure of the remaining parts of this paper is outlined as follows. In section 2, the theoretical model and governing equations are presented. In section 3, we analyzed the nonlinear generation of ZS in detail. In section 4, we incorporated the description of beat-driven ZS into the existing linear dispersion relation to investigate the ‘nonlinear’ effects on ITG linear stability. In section 5, analytical and numerical results are discussed both in the short- and long-wavelength regimes. Finally, section 6 concludes with a summary and discussion of this work.

2. Theoretical model

For simplicity and clarity of discussion, we consider a low- β tokamak that has a large aspect ratio and concentric circular magnetic surfaces, with the equilibrium magnetic field being given by $\mathbf{B} = B_0 [\mathbf{e}_\zeta / (1 + \varepsilon \cos \theta) + \varepsilon \mathbf{e}_\theta / q]$. Here, $\varepsilon \equiv r/R < 1$ represents the inverse aspect ratio, where R denotes the major radius of the tokamak and r represents the minor ones, $\beta \equiv 8\pi P/B_0^2 \sim \mathcal{O}(\varepsilon^2) \ll 1$ is the ratio between plasma and magnetic pressure, ζ is identified as the toroidal angle and θ as the poloidal angle respectively, and $q \equiv rB_\zeta/(RB_\theta)$ serves as the safety factor. To simplify the analysis while focusing on the main physics, the equilibrium distribution functions of thermal plasmas are taken to be local Maxwellian, while trapped particle effects are systematically neglected. $\delta\phi$ and $\delta A_\parallel$ are taken as the field variables, where $\delta\phi$ represents the scalar potential, while $\delta A_\parallel$ denotes the parallel component of the vector potential, i.e. $\delta \mathbf{A} \simeq \delta A_\parallel \mathbf{b}$ and $\mathbf{b} \equiv \mathbf{B}/B$ is the unit vector along the equilibrium magnetic field. Another field variable, $\delta\psi \equiv \omega \delta A_\parallel / (ck_\parallel)$, is also adopted in the case of $n \neq 0$

TAEs, and within the ideal MHD limit, the equality $\delta\psi = \delta\phi$ is obtained.

In gyrokinetic theory, the perturbed distribution function, δf_s , with the subscript $s = e, i$ standing for the particle species, can be denoted as

$$\delta f_s = -\left(\frac{e}{T}\right)_s \delta\phi F_{Ms} + \exp(-\rho_s \cdot \nabla) \delta H_s, \quad (1)$$

and its non-adiabatic component δH_s obeys the nonlinear gyrokinetic equation [38]

$$\begin{aligned} (\partial_t + v_{\parallel} \partial_l + i\omega_{Ds}) \delta H_s = & -i \left(\frac{e}{T}\right)_s (\omega - \omega_{*s}^t) F_{Ms} J_k \delta L_k \\ & - \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \Lambda_{k'}^{k'} J_{k'} \delta L_{k'} \delta H_{k''}. \end{aligned} \quad (2)$$

Here, F_{Ms} is the Maxwellian distribution representing equilibrium thermal particle distribution, $\rho_s = \mathbf{b} \times \mathbf{v}/\omega_{cs}$ is the Larmor radius with $\omega_{cs} \equiv eB_0/(m_s c)$ being the cyclotron frequency, l is the arc length along the equilibrium magnetic field line, $\omega_{Ds} = 2\hat{\omega}_{ds}C$ represents the magnetic drift frequency, with $\hat{\omega}_{ds} \equiv \omega_{ds}(x_{\perp}^2/2 + x_{\parallel}^2)$, $\omega_{ds} \equiv k_{\theta} c T_s/(e_s B_0 R)$, $x_{\perp} = v_{\perp}/v_{ts}$ and $x_{\parallel} = v_{\parallel}/v_{ts}$ being the particle perpendicular/parallel velocities normalized by thermal velocity $v_{ts} \equiv \sqrt{2T_s/m_s}$, respectively, and $C \equiv \cos\theta - \sin\theta k_r/k_{\theta}$ is related to the magnetic curvature, with k_r and $k_{\theta} = -m_0/r_0$ being the radial/poloidal wavenumbers. Furthermore, $\omega_{*s}^t \equiv \omega_{*s}(1 + \eta_s(v^2/v_{ts}^2 - 3/2))$ denotes the diamagnetic drift frequency related to plasma nonuniformity, with $\omega_{*s} = k_{\theta} c T_s/(e_s B_0 L_n)$, $\eta_s = L_{ns}/L_{Ts}$, where $L_n \equiv -N/(\partial N/\partial r)$, $L_T \equiv -T/(\partial T/\partial r)$ are the characteristic lengths of density and temperature nonuniformities, respectively, $J_k = J_0(k_{\perp} \rho_s)$ is the Bessel function and $k_{\perp} = \sqrt{k_r^2 + k_{\theta}^2}$ is perpendicular wavenumber. Furthermore, $\Lambda_{k'}^{k'} \equiv (c/B_0) \mathbf{b} \cdot (\mathbf{k}'' \times \mathbf{k}')$, with $\sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \Lambda_{k'}^{k'}$, denoting the selection rule for the frequency and wavenumber matching conditions of the nonlinear mode coupling, $\delta L_k \equiv \delta\phi_k - k_{\parallel} v_{\parallel} \delta\psi_k/\omega_k$ is the scalar potential in the guiding-center moving frame, and other notations are standard. In the following analysis, the non-adiabatic particle response δH_s can be separated into linear and nonlinear components, i.e. $\delta H \equiv \delta H^L + \delta H^{NL}$, with the superscripts 'L' and 'NL' denoting the linear and nonlinear components, respectively, and can be derived perturbatively assuming $\delta H^{NL} \ll \delta H^L$.

In the $\beta \ll 1$ limit with negligible magnetic compression, the charge quasi-neutrality condition

$$\frac{Ne^2}{T_i} \left(1 + \frac{1}{\tau}\right) \delta\phi_k = \sum_s \langle e_s J_k \delta H_k \rangle, \quad (3)$$

can be used to close the nonlinear gyrokinetic equation and parallel Ampere's law

$$-\frac{c}{4\pi} \nabla_{\perp}^2 \delta A_{\parallel} = \delta J_{\parallel} = \langle -ev_{\parallel} \delta f_e \rangle. \quad (4)$$

Here, terms on the left hand side of equation (3) are contributions from adiabatic responses of ion and electron, respectively, $\tau \equiv T_e/T_i$ is the electron to ion temperature ratio, and $\langle \dots \rangle$ represents velocity space integration.

We now consider the effects of indirect modulation of ITG stability by finite amplitude TAE, mediated by beat-driven ZS. More specifically, this is a two-step process, with the first process being a finite amplitude TAE $\Omega_0 \equiv \Omega_0(\omega_0, \mathbf{k}_0)$ couples with its complex conjugate $\Omega_0^* \equiv \Omega_0^*(-\omega_0, -\mathbf{k}_0)$ to generate ZS $\Omega_Z \equiv \Omega_Z(\omega_Z, \mathbf{k}_Z)$ with $\omega_Z \approx 0$, $k_{\parallel Z} = 0$, $k_{\theta Z} = 0$, and finite k_{rZ} , which is analyzed in section 3. In the second process, the beat-driven ZS affects the linear stability of ITG $\Omega_I \equiv \Omega_I(\omega_I, \mathbf{k}_I)$, and is analyzed in section 4. Here, the subscripts 0, Z and I denote quantities associated with the TAE, ZS and ITG, respectively. For DWs, especially ITG, with the most unstable modes characterized by high poloidal mode numbers, the distance between two neighboring mode rational surfaces is generally, much smaller than the scale of equilibrium profile variation. Thus, the well-known ballooning-mode decomposition [39] can be adopted to decompose the two-dimensional eigenvalue problem into two coupled one-dimensional eigenvalue problems, i.e.,

$$\delta\phi_I = A_I e^{-in\zeta - i\omega t} \sum_j \Phi_I(x-j) e^{i(m_0+j)\theta} + c.c. \quad (5)$$

Here, A_I is the radial envelope of ITG, n and $(m = m_0 + j)$ stand for the toroidal and poloidal mode numbers, respectively, where m_0 serves as its reference value fulfilling the equation $nq(r_0) = m_0$, and r_0 denotes the plasma radial position at which the ITG is assumed to be localized. Furthermore, $x = nq - m_0 \simeq nq'(r - r_0)$, $|j| \ll m_0$ is an integer, and $\Phi_I(x-j)$ represents the fine radial structure caused by finite $k_{\parallel}$, with $\int |\Phi_I|^2 dx = 1$ being normalization condition. In the present analysis, however, we will focus on the local stability of ITG, as the ZS driven by TAE is expected to have a larger radial scale than ITG [11]. The global problem with the radial envelope modulation will be analyzed in a future publication.

3. Nonlinear ZS generation

In this section, based on the gyrokinetic theoretic framework, i.e. equations (2)–(4), an investigation of the nonlinear generation of ZS by TAEs self-beating is carried out. Briefly speaking, the particle responses to ZS due to TAE self-coupling (i.e. PSZS) are derived from the nonlinear gyrokinetic equation, which are then substituted into the quasi-neutrality condition and parallel Ampere's law to derive the zonal scalar and vector potential response, which will then be used to study the TAE effects on ITG via ZS mediation in section 4.

We start from the linear particle response to the finite amplitude TAE Ω_0 , which will be used in the nonlinear term of the gyrokinetic equation later. For massless-electron with $|k_{\parallel} v_{te}/\omega_0| \gg 1$, $|k_{\perp} \rho_e| \ll 1$ and $J_0(k_{\perp} \rho_e) \sim 1$ satisfied, the linear electron response to Ω_0 can be straightforwardly derived as

$$\delta H_{0,e}^L \simeq -\frac{e}{T_e} \left(1 - \frac{\omega_{*e,0}^t}{\omega_0}\right) F_{Me} \delta \psi_0. \quad (6)$$

Meanwhile, for ions with $|k_{\parallel} v_{ti}/\omega_0| \ll 1$, we have

$$\delta H_{0,i}^L \simeq \frac{e}{T_i} \left(1 - \frac{\omega_{*i,0}^t}{\omega_0}\right) F_{Mi} J_0 \delta \phi_0. \quad (7)$$

Substituting above expressions into the quasi-neutrality condition and vorticity equation [7], the dispersion relation of TAE can be obtained. In our analysis, however, focusing on TAE effects on ITG stability, we do not need a detailed expression of the TAE dispersion relation.

The linear and nonlinear particle responses to ZS can be derived by transforming equation (2) into drift orbit center coordinate, i.e. taking $\delta H_{Z,s} \equiv \delta H_{dZ,s} \exp(i\lambda_Z)$, with $\lambda_Z \equiv \hat{\lambda}_{dZ} \cos \theta$ satisfying $\omega_{tr} \partial_{\theta} \lambda_Z + \omega_{Drs} = 0$, and $\hat{\lambda}_{dZ} \equiv -2\hat{\omega}_{dr}/\omega_{tr}$ being the normalized drift orbit width. Here, $\omega_{tr} \equiv v_{\parallel}/qR$ is the transit frequency, $\omega_{Drs} \equiv -2\hat{\omega}_{drs} \sin \theta$, and $\hat{\omega}_{drs} \equiv k_r c T_s (x_{\perp}^2/2 + x_{\parallel}^2)/(e_s B_0 R)$. Substituting $\delta H_{Z,s}$ into equation (2), we have

$$(\partial_t + \omega_{tr} \partial_{\theta}) \delta H_{dZ} = -i \left(\frac{e}{T}\right)_s e^{-i\lambda_Z} \omega_Z F_{Ms} J_Z \left(\delta \phi - \frac{v_{\parallel}}{c} \delta A\right)_Z - \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} e^{-i\lambda_Z} \Lambda_{k',J_{k'}}^{\mathbf{k}'} \delta L_{k'} \delta H_{k'}^{\mathbf{k}'} \quad (8)$$

Noting the $v_{ts}/(qR) \gg \omega_Z$ ordering for both electrons and ions and taking the dominant flux surface averaged quantities $\overline{(\cdots)} \equiv \int_0^{2\pi} (\cdots) d\theta / (2\pi)$, the linear particle response to Ω_Z can be straightforwardly derived from the linear term of equation (8) as

$$\overline{\delta H_{Z,e}^L} \simeq -\frac{e}{T_e} F_{Me} \left(\delta \phi - \frac{v_{\parallel}}{c} \delta A\right)_Z, \quad (9)$$

$$\overline{\delta H_{Z,i}^L} = \frac{e}{T_i} F_{Mi} J_Z |\theta_Z|^2 \left(\delta \phi - \frac{v_{\parallel}}{c} \delta A\right)_Z, \quad (10)$$

where $\theta_Z \equiv \overline{e^{-i\lambda_Z}}$. Meanwhile, noting that the linear particle responses to TAE are given in equations (6) and (7), the nonlinear particle response to Ω_Z can be derived as

$$\overline{\delta H_{Z,e}^{NL}} = -\frac{c}{B_0} k_{\theta 0} \frac{e}{T_e} F_{Me} \left(\frac{\omega_{*e,0}^t}{\omega_0^2} - \frac{k_{\parallel 0} v_{\parallel}}{\omega_0^2}\right) \partial_r |\delta \phi_0|^2, \quad (11)$$

$$\overline{\delta H_{Z,i}^{NL}} = \frac{c}{B_0} |\theta_Z|^2 k_{\theta 0} \frac{e}{T_i} J_0^2 F_{Mi} \frac{\omega_{*i,0}^t}{\omega_0^2} \partial_r |\delta \phi_0|^2, \quad (12)$$

where $\partial_r = ik_{rZ}$ is the operator for radial derivative. It is reasonable to conclude from equations (11) and (12) that, $\partial_r |\delta \phi_0|^2 \sim ik_{rZ} |\delta \phi_0|^2$, corresponding to the fine scale ZS generation by weakly/moderately ballooning TAE [11]. Substituting the particle responses to Ω_Z , i.e. equations (9)–(12) into equation (3), a nonlinear equation for the zonal scalar

potential generation by TAE in nonuniform plasmas can be derived as

$$\delta \phi_Z = \frac{c}{B_0} k_{\theta 0} \frac{\omega_{*i,0}}{\omega_0^2} \left[\frac{\eta_i}{\chi_{iZ}} \left\langle \frac{F_{Mi}}{n_0} J_Z |\theta_Z|^2 J_0^2 \right\rangle \times \left(\frac{v^2}{v_{ti}^2} - \frac{3}{2} \right) \right] \partial_r |\delta \phi_0|^2. \quad (13)$$

Here, $\chi_{iZ} \equiv 1 - \left\langle F_{Mi}/N J_Z^2 |\theta_Z|^2 \right\rangle \approx 1.6 k_{rZ}^2 \rho_{ti}^2 q^2 / (2\sqrt{\varepsilon})$ is the well known neoclassical inertia enhancement dominated by the trapped particle contribution [27], and $\rho_{ti} \equiv v_{ti}/\omega_{ci}$.

If we neglect the FLR effects by taking $J_Z(k_{rZ} \rho_i) \simeq 1$, noting that, $\left\langle F_{Mi}/N |\theta_Z|^2 (v^2/v_{ti}^2 - 3/2) \right\rangle = -\chi_{iZ}$, equation (13) can be explicitly written as

$$\delta \phi_Z = -\frac{c}{B_0} k_{\theta 0} \frac{\omega_{*pi,0}}{\omega_0^2} \partial_r |\delta \phi_0|^2. \quad (14)$$

Here, $\omega_{*pi} \equiv \omega_{*i} + \omega_{*Ti}$ with $\omega_{*Ti} = \eta_i \omega_{*i}$. Equation (14) describes ZF beat-driven by TAE, demonstrating that the growth rate of the beat-driven zonal scalar potential is twice that of the TAE instantaneous growth rate, and its magnitude is proportional to the amplitude of the TAE, which is a common trait of the beat-driven process [35, 40, 41]. Furthermore, the beat-driven process has no threshold, which is distinctly different from the spontaneous excitation process generated by the modulational instability of TAE, where a sufficiently large TAE amplitude is required to overcome the threshold caused by frequency mismatch [11]. And that is precisely the reason for the common observation of the beat-driven process in simulation works [40–43].

Meanwhile, the equation for ZC generation can be derived from the parallel component of Ampere's law [31]. One obtains from the zonal component of equation (4),

$$\delta A_{\parallel Z} \simeq -\frac{c^2}{B_0} k_{\theta 0} \frac{k_{\parallel 0}}{\omega_0^2} \partial_r |\delta \phi_0|^2, \quad (15)$$

with the assumption of $k_{\perp}^2 c^2 / \omega_{pe}^2 \ll 1$, and $\omega_{pe}^2 = 4\pi n_0 e^2 / m_e$ being the electron plasma frequency. Equations (14) and (15), together with the nonlinear particle response to ZS given by equations (11) and (12) (i.e. PSZS), will be used to study TAE effects on ITG via ZS mediation in section 4.

4. Effects of beat-driven ZS on DW linear stability

Next, we consider the effects of beat-driven ZS on ITG linear stability. For simplicity, we consider the ITG to be electrostatic, and the analysis closely conforms to that of [37]. The ITG equation is derived from the quasi-neutrality condition, i.e. equation (3), with the particle responses derived from the nonlinear gyrokinetic equation.

For ITG with $k_{\parallel} v_{te} \gg \omega \sim \omega_{*i} \gg \omega_{Ds}$, the linear electron response to ITG is adiabatic, i.e.,

$$\delta H_{I,e}^L \approx 0;$$

and for ion with $\omega \gg k_{\parallel} v_{ti}$, the linear ion responses to ITG can be derived as

$$\delta H_{I,i}^L \simeq \frac{e}{T_i} \left(1 - \frac{\omega_{*i}^t}{\omega} \right) \left(1 + \frac{v_{\parallel} k_{\parallel}}{\omega} + \frac{v_{\parallel}^2 k_{\parallel}^2}{\omega^2} + \frac{\omega_{Di}}{\omega} \right) F_{Mi} J_0 \delta \phi_I.$$

Here, we have adopted the fluid-ion approximation to simplify the analysis, and thereby, more clearly illustrate the effects of ZS on ITG stabilizing. Substituting the linear response of particles to ITG into the quasi-neutrality condition, the linear ITG equation can be derived as

$$\varepsilon_I \delta \phi_I = 0, \quad (16)$$

with

$$\varepsilon_I = 1 - \frac{\omega_{*e}}{\omega} + \tau \left(1 - \frac{\omega_{*pi}}{\omega} \right) \left(b_{\perp} - \frac{k_{\parallel}^2 v_{ti}^2}{2\tau\omega^2} - \frac{2\omega_{di}C}{\omega} \right) \quad (17)$$

being the linear ITG dielectric operator, and $b_{\perp} = k_{\perp}^2 \rho_i^2 / 2$. The contributions of beat-driven ZS, enter through the formally ‘nonlinear’ terms, including scattering by zonal field structure (i.e. $\delta \phi_Z$ and $\delta A_{\parallel Z}$) via $J_Z \delta L_Z \delta H_I$ and nonlinear modification of equilibrium by PSZS via $J_I \delta \phi_I \delta H_Z^{\text{NL}}$ in the nonlinear gyrokinetic equation

$$(\partial_t + v_{\parallel} \partial_t + i\omega_D) \delta H_I^{\text{NL}} = \Lambda [J_I \delta \phi_I (\delta H_Z^L + \delta H_Z^{\text{NL}}) - J_Z \delta L_Z \delta H_I^L], \quad (18)$$

with $\Lambda = -ck_{rZ}k_{\theta I}/B_0$. Noting the $k_{\parallel} v_{te} \gg \omega \gg k_{\parallel} v_{ti}$ ordering again, we have

$$\delta H_{I,e}^{\text{NL}} \simeq 0, \quad (19)$$

and

$$\delta H_{I,i}^{\text{NL}} = \frac{e}{T_i} F_{Mi} \left[\left(|\theta_Z|^2 - 1 + \frac{\omega_{*i}^t}{\omega} \right) \frac{\omega_{*pi}}{\omega} - \frac{\omega_{*i}^t}{\omega} |\theta_Z|^2 \right] \delta \hat{\phi}_0^2 \delta \phi_I. \quad (20)$$

Here, $\delta \hat{\phi}_0^2 = |ck_{\theta 0}/(B_0 \omega_0)|^2 \partial_r^2 |\delta \phi_0|^2$. Note that $\delta \hat{\phi}_0$ is related to the normalized Doppler shift induced by the radial electric field of TAE and is also related to the beat-driven ZF induced shearing rate. In deriving equations (19) and (20), we neglected the FLR effects by taking $J_Z(k_{rZ} \rho_i) \simeq 1$, and the expressions of the beat-driven ZS given by equations (11), (12), (14) and (15) are used. Substituting the particle responses of ITG into equation (3) yields the equation that describes the nonlinear modulation of the beat-driven ZS on ITG:

$$\frac{n_0 e^2}{T_e} \left[\varepsilon_I + \tau \frac{\omega_{*i}}{\omega} \left(1 - \frac{\omega_{*pi}}{\omega} \right) \delta \hat{\phi}_0^2 \right] \delta \phi_I = 0. \quad (21)$$

Combining equations (17) and (21), one then has the ITG dispersion relation in the WKB limit

$$\left[\frac{\omega}{\tau(\omega - \omega_{*pi})} + \frac{\omega_{*i}}{\omega - \omega_{*pi}} + b_{\perp} - \frac{k_{\parallel}^2 v_{ti}^2}{2\omega^2} - \frac{2\omega_{di}C}{\omega} + \frac{\omega_{*i}}{\omega} \delta \hat{\phi}_0^2 \right] \delta \phi_I = 0. \quad (22)$$

Here, operators ω , $b_{\perp}$ and $k_{\parallel}$ without subscripts $k=0, Z$ represent ITG. The foremost five terms of equation (22) compose the linear ITG dispersion relation, which deviates slightly from that in [37], primarily due to the inclusion of both density and temperature nonuniformities in the present analysis, while [37] took the flat density gradient limit to focus on the effects of ITG, following the early analysis of [44]. Moreover, the first four terms correspond to the adiabatic electron response, ion $\mathbf{E} \times \mathbf{B}$ drift, the FLR effect (polarization), and the parallel compressibility, respectively. The fifth term accounts for the magnetic drift, which is unique to toroidal configurations and leads to the coupling of adjacent poloidal harmonics. The last term introduces a nonlinear modification attributed to the beat-driven ZS, and is the main novel contribution of the present work. Additionally, in this context, the final term represents a nonlinear correction that takes into account the combined contributions of ZF, ZC, and PSZS. The contribution of each of the three components to the nonlinear ITG dispersion relation will be presented in the [appendix](#).

Noting $k_{\perp}^2 = k_{\theta}^2 - \partial^2/\partial r^2$, and introducing the Fourier transform $\Phi(\eta) = \int \Phi(nq - m) \exp(-i\eta(nq - m)) d(nq - m)$ with $nq - m$ being the normalized distance to the mode rational surface and $\Phi(nq - m)$ being the poloidal harmonic of the ITG, the ITG eigenmode equation in ballooning space can be derived as

$$\frac{d^2 \Phi(\eta)}{d\eta^2} + q^2 \Omega^2 b \left[- \frac{\tau}{\left(1 + \tau \Omega \varepsilon_{pi}^{1/2} \right) (1 + \eta_i) \varepsilon_{pi}^{1/2}} + \frac{\tau \Omega}{1 + \tau \Omega \varepsilon_{pi}^{1/2}} + b(1 + \hat{s}^2 \eta^2) + \frac{2}{\Omega} (\cos \eta + \hat{s} \eta \sin \eta) \right. \\ \left. - \frac{1}{\Omega \varepsilon_{pi} (1 + \eta_i)} \delta \hat{\phi}_0^2 \right] \Phi(\eta) = 0, \quad (23)$$

where $\hat{s} \equiv r(\partial q/\partial r)/q$ denotes the magnetic shear, $\Omega \equiv \omega/(\tau \sqrt{\omega_{*pi} \omega_{di}})$, $b \equiv \tau b_{\theta}/\sqrt{\varepsilon_{pi}}$, and $\varepsilon_{pi} \equiv L_{pi}/R$. Equation (23) is the paradigmatic one-dimensional eigenvalue equation with a potential well contributed by two components: a slowly varying parabolic well stemming from the FLR effect ($1 + \hat{s}^2 \eta^2$), and a rapidly oscillating periodic well due to magnetic curvature ($\cos \eta + \hat{s} \eta \sin \eta$), while the beat-driven ZS by TAE enters the ITG eigenmode equation by modifying the potential well depth while not affecting the parity or periodicity. These distinct potential wells result in two principal ITG eigenmode branches: the slab branch sensitive to the slow parabolic variation ($1 + \hat{s}^2 \eta^2$), and the toroidal branch due to the rapid periodic fluctuations ($\cos \eta + \hat{s} \eta \sin \eta$). In section 5, following the analysis of [45, 46], we assess the impact of ZS on ITG characteristics, including frequency, growth rate, and mode structure, in the short- and long-wavelength regimes, respectively. The analytical results are then compared with the numerical solution of equation (23) using the eigenmatrix method. In equation (23), we also note that $\delta \hat{\phi}_0^2 = |ck_{\theta 0}/(B_0 \omega_0)|^2 \partial_r^2 |\delta \phi_0|^2 \sim -|k_{rZ}/k_{\parallel 0}|^2 |\delta B_r/B_0|^2$, assuming the TAE is close to the ideal MHD condition. Taking typical tokamak parameters, i.e. $k_{rZ} \sim 1/\rho_E$, $k_{\parallel 0} \sim 1/(2qR)$,

and $|\delta B_r/B_0|^2 \leq 2.5 \times 10^{-7}$ for TAE fluctuations level expected in reactors [47], we then find $\delta\hat{\phi}_0^2 \sim \mathcal{O}(10^{-1})$, and $e\delta\phi_Z/T_i \leq 5 \times 10^{-4}$.

5. Analytical and numerical results

In this section, following the analyses in [45, 46], we solve equation (23) analytically in short- and long-wavelength limits, $b \sim \mathcal{O}(1)$ and $b \ll 1$, respectively. The analytical results are then compared with the general numerical solutions of equation (23) using matrix eigenvalue method in the corresponding limits. Based on these results, the impact of TAE-driven ZS on ITG stability is quantitatively analyzed.

5.1. Short-wavelength limit

In the short-wavelength limit with $b \sim \mathcal{O}(1)$, the parallel mode structure is strongly localized around $\eta = 0$ in the ballooning space, which allows us to perform a Taylor expansion of the potential function around $\eta = 0$, and reduces equation (23) into a Weber equation:

$$\begin{aligned} \frac{d^2\Phi(\eta)}{d\eta^2} + q^2\Omega^2b \left[-\frac{\tau}{\left(1 + \tau\Omega\varepsilon_{pi}^{1/2}\right)(1 + \eta_i)\varepsilon_{pi}^{1/2}} \right. \\ \left. + \frac{\tau\Omega}{1 + \tau\Omega\varepsilon_{pi}^{1/2}} + b + \frac{2}{\Omega} - \frac{1}{\Omega\varepsilon_{pi}(1 + \eta_i)}\delta\hat{\phi}_0^2 \right. \\ \left. + \left(b\hat{s}^2 + \frac{2\hat{s}-1}{\Omega}\right)\eta^2 \right] \Phi(\eta) = 0. \end{aligned} \quad (24)$$

Then, the solutions of ITG can readily be obtained from the well-known eigenfunctions of Weber equation, and among them, the most unstable mode or, more precisely, the ground-state eigenfunction is given by $\delta\phi = \exp(-\alpha\eta^2)$, with

$$\begin{aligned} \alpha \equiv \frac{q^2\Omega^2b}{2} \left[-\frac{\tau}{\left(1 + \tau\Omega\varepsilon_{pi}^{1/2}\right)(1 + \eta_i)\varepsilon_{pi}^{1/2}} \right. \\ \left. + \frac{\tau\Omega}{1 + \tau\Omega\varepsilon_{pi}^{1/2}} + b + \frac{2}{\Omega} - \frac{1}{\Omega\varepsilon_{pi}(1 + \eta_i)}\delta\hat{\phi}_0^2 \right]. \end{aligned} \quad (25)$$

Notice that the real part of α is proportional to b , which means that in the ballooning mode space, the half-width of the most unstable mode is inversely proportional to $1/\sqrt{b}$, and, therefore, the half-width in real space is directly proportional to $\sqrt{b}$. Consequently, under the parameter condition where $b \sim \mathcal{O}(1)$, the width of the ground-state eigenmode meets the criteria for the strong coupling approximation. Furthermore,

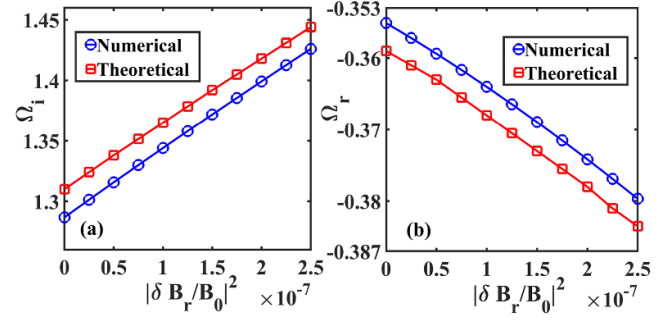


Figure 1. Dependence of the ITG growth rate (a) and real frequency (b) on the amplitude of TAE, which are normalized by $\tau\sqrt{\omega_{*pi}\omega_{di}}$. The blue represents numerical results of equation (23) with $b = 1$ limit, while the red represents the theoretical given by equation (26). Here, $\eta_i = 2$, $\varepsilon_{pi} = 0.06$, $q = 1$, $\tau = 1$ and $\hat{s} = 1$.

the corresponding dispersion relation is

$$\begin{aligned} q^2\Omega^2b \left[-\frac{\tau}{\left(1 + \tau\Omega\varepsilon_{pi}^{1/2}\right)(1 + \eta_i)\varepsilon_{pi}^{1/2}} \right. \\ \left. + \frac{\tau\Omega}{1 + \tau\Omega\varepsilon_{pi}^{1/2}} + b + \frac{2}{\Omega} - \frac{1}{\Omega\varepsilon_{pi}(1 + \eta_i)}\delta\hat{\phi}_0^2 \right]^2 \\ + \left(b\hat{s}^2 + \frac{2\hat{s}-1}{\Omega}\right) = 0. \end{aligned} \quad (26)$$

From equation (26), we can analytically derive the linear growth rate and real frequency of the ground-state eigenmode under the strong coupling approximation, which are then compared with the numerical solutions obtained from equation (23) at $b = 1$, as shown in figure 1.

There is a remarkable consistency between the analytical and numerical data. It is found that, the ITG real frequency as well as growth rate change only slightly with TAE amplitude with the typical tokamak parameter regime, i.e. $|\delta B_r/B_0|^2 \leq 2.5 \times 10^{-7}$. More importantly, the ITG growth rate increases with TAE amplitude, which suggests that the ZS beat driven by TAE has only a weakly destabilizing effect on ITG stability, contrary to the general speculation. In particular, the influence of ZS on ITG in our work is significantly less pronounced than the results reported in [37], where effects of radial electric field from electro-static ZFZF on ITG stability are investigated. This discrepancy primarily stems from the coefficient of the nonlinear term in our equation (23) being substantially smaller than the corresponding coefficient in equation (7) of [37]. In fact, even at the maximum TAE amplitude adopted, the amplitude of ZS beat-driven by TAE is also quite small, i.e. $e\delta\phi_Z/T_i \leq 5 \times 10^{-4}$, so it is reasonable that the impact of TAE beat-driven ZS on the ITG linear stability is correspondingly weak. If the coefficient of the nonlinear term is artificially enlarged by a factor of five, as depicted by the magenta dotted line in figure 2, the destabilizing effect of TAE beat-driven ZS on ITG becomes remarkably evident. In addition,

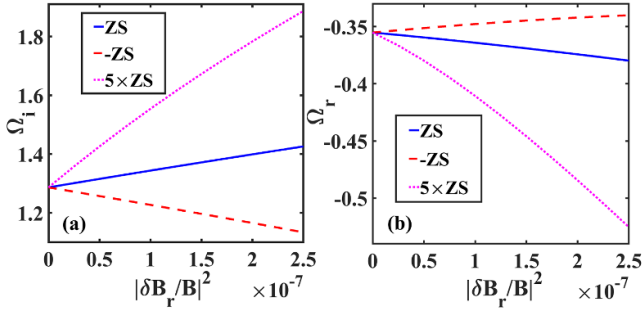


Figure 2. Dependence of the ITG growth rate (a) and real frequency (b) on the amplitude of TAE with different ZS values, which are normalized by $\tau\sqrt{\omega_{*pi}\omega_{di}}$. Here, $\eta_i = 2$, $\varepsilon_{pi} = 0.06$, $q = 1$, $\tau = 1$, $\hat{s} = 1$ and $b = 1$.

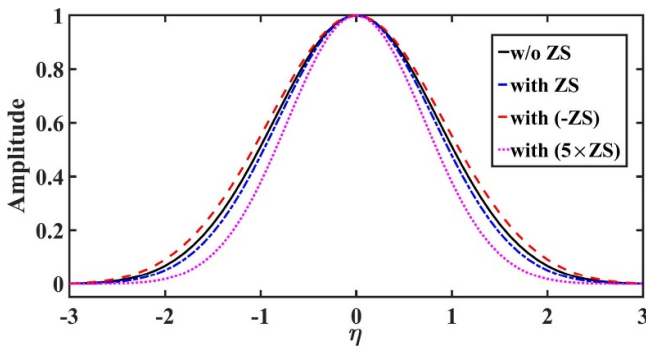


Figure 3. The mode structures of the ground-state eigenmode with different ZS values. Here, $\eta_i = 2$, $\varepsilon_{pi} = 0.06$, $q = 1$, $\tau = 1$, $\hat{s} = 1$ and $b = 1$.

in the present work, the linear growth rate of ITG increases with TAE amplitude, which is in contrast to the numerical results in [37]. The primary reason is attributed to the difference in the sign of the nonlinear terms, which is opposite to that in [37]. As the sign of the nonlinear term in equation (23) is artificially reversed, as illustrated by the long red dashed line in figure 2, the ITG growth rate decreases with TAE amplitude, similar to the results of [37]. However, the underlying physical mechanisms remain elusive and are worthy of further investigation.

In the ballooning space, the impact of TAE-driven-ZS on the corresponding mode structures of the ground-state eigenmode are shown in figure 3, where the blue dot-dash curve represents the case with $|\delta B_r/B_0|^2 = 2.5 \times 10^{-7}$, red dashed and magenta dotted curves represent the cases with the nonlinear term artificially changed sign and increased by 5 times, respectively; and the black curve represents the linear mode structure. Clearly, the TAE beat-driven ZS reduces the half-width of the most unstable ITG mode structure. However, even at the maximum amplitude of TAE, the impact of the TAE thermally driven ZS on the ITG mode structure remains minimal. The primary reason is that the TAE beat-driven ZS slightly reduces the depth of the potential well, thereby decreasing the distance between the turning points of the

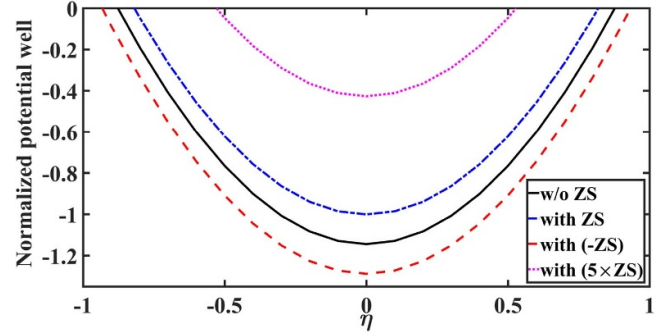


Figure 4. Potential wells with different values of ZS. Here, $\eta_i = 2$, $\varepsilon_{pi} = 0.06$, $q = 1$, $\tau = 1$, $\hat{s} = 1$ and $b = 1$.

potential well and localizing the ITG mode within a narrower range of stronger drive at the bad curvature region, as shown in figure 4, and this is confirmed by the magenta dotted curve and red dashed curve, where the nonlinear term is artificially changed sign or enlarged by a factor. Additionally, the mode structure peaks at $\eta = 0$, and the even symmetry remains unbroken, a result of the even-symmetric modulation introduced by the beat-driven ZS.

5.2. Long-wavelength limit

As previously mentioned, the strong coupling approximation requires $b \geq \mathcal{O}(1)$, which is a rather stringent condition. In more general scenarios, the long wavelength limit with $b \ll 1$ is satisfied, where the radial width of the ITG poloidal harmonics gradually decreases and the coupling between adjacent poloidal harmonics becomes weak. At this point, in addition to the toroidal branch, there also exists a slab branch that primarily responds to the slowly varying parabolic potential well.

In this work, we are more concerned with the toroidal branch [46], but neglect the solution process of the slab branch. Following the theoretical approach in [46], the mode structure expression for the toroidal branch is defined as $\Phi(\eta) = S(\eta_1) \sin \eta/2 + C(\eta_1) \cos \eta/2$, where $\eta_1 \equiv \hat{\varepsilon}\eta$ and $\hat{\varepsilon} = b^{1/3}$ represent slow variation of η . Substituting this mode structure expression into equation (23) and further separating the coefficients of $\sin \eta/2$ and $\cos \eta/2$, the eigenmode equation can be derived as

$$\frac{dS}{d\eta_1} + \left[\frac{q^2 b^{2/3} \Omega^3 \tau}{1 + \tau \Omega \varepsilon_{pi}^{1/2}} - \frac{q^2 b^{2/3} \Omega^2 \tau}{(1 + \tau \Omega \varepsilon_{pi}^{1/2})(1 + \eta_i) \varepsilon_{pi}^{1/2}} - \frac{1}{4b^{1/3}} - \frac{q^2 b^{2/3} \Omega}{\varepsilon_{pi}(1 + \eta_i)} \delta \hat{\phi}_0^2 \right] C + q^2 \Omega b^{1/3} \hat{s} \eta_1 S = 0, \quad (27)$$

$$\frac{dC}{d\eta_1} - \left[\frac{q^2 b^{2/3} \Omega^3 \tau}{1 + \tau \Omega \varepsilon_{pi}^{1/2}} - \frac{q^2 b^{2/3} \Omega^2 \tau}{(1 + \tau \Omega \varepsilon_{pi}^{1/2})(1 + \eta_i) \varepsilon_{pi}^{1/2}} - \frac{1}{4b^{1/3}} - \frac{q^2 b^{2/3} \Omega}{\varepsilon_{pi}(1 + \eta_i)} \delta \hat{\phi}_0^2 \right] S - q^2 \Omega b^{1/3} \hat{s} \eta_1 C = 0. \quad (28)$$

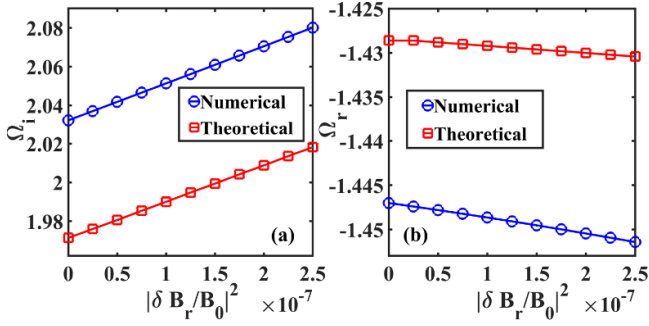


Figure 5. Dependence of the ITG growth rate (a) and real frequency (b) on the amplitude of TAE, which are normalized by $\tau\sqrt{\omega_{pi}\omega_{di}}$. The blue represent numerical results of equation (23) with $b = 0.01$ limit, while the red represent the theoretical given by equation (30). Here, $\eta_i = 2$, $\varepsilon_{pi} = 0.06$, $q = 1$, $\tau = 1$ and $\hat{s} = 1$.

Combining these two equations yields a Weber equation of S :

$$\frac{d^2 S}{d\eta_i^2} - \left[\frac{q^2 b^{2/3} \Omega^3 \tau}{1 + \tau \Omega \varepsilon_{pi}^{1/2}} - \frac{q^2 b^{2/3} \Omega^2 \tau}{(1 + \tau \Omega \varepsilon_{pi}^{1/2})(1 + \eta_i) \varepsilon_{pi}^{1/2}} - \frac{1}{4b^{1/3}} \right. \\ \left. - \frac{q^2 b^{2/3} \Omega}{\varepsilon_{pi}(1 + \eta_i)} \delta \hat{\phi}_0^2 \right]^2 S + q^2 \Omega b^{1/3} \hat{s} S \\ - q^4 \Omega^2 b^{2/3} \hat{s}^2 \eta_i^2 S = 0, \quad (29)$$

and the dispersion relation for the ground state of the toroidal branch is given by

$$\Omega^3 - \Omega^2 \left[\frac{1}{(1 + \eta_i) \varepsilon_{pi}^{1/2}} - \frac{1}{(1 + \eta_i) \varepsilon_{pi}^{1/2}} \delta \hat{\phi}_0^2 \right. \\ \left. \times \left(1 + \frac{1}{\tau \Omega \varepsilon_{pi}^{1/2}} \right) \right] = \frac{1}{4bq^2 \tau} (1 + \tau \Omega \varepsilon_{pi}^{1/2}). \quad (30)$$

Equation (30) is observed to be similar to the linear result in [46], with the additional term proportional to $\delta \hat{\phi}_0^2$ originating from the contribution of beat-driven ZS.

Following the same procedures as in the short-wavelength limit, we quantitatively analyze the impact of TAE-driven-ZS on the linear stability of ITG ground-state eigenmodes under the parameter condition $b = 0.01$, with results shown in figure 5. Analytical and numerical results indicate that in the long-wavelength limit, the growth rate as well as the real frequency of the toroidal branch vary in a similar manner to the results of the short-wavelength limit. That is, the ITG growth rate slightly increases with TAE amplitude, indicating that ZS beat-driven by TAE has only a weakly destabilizing effect on ITG stability due to the relatively small amplitude of the ZS, as well as its phase. The analysis methods are similar to those used in the short-wavelength limit, and thus, will not be repeated here. The mode structure in the long-wavelength limit is presented in figure 6, which indicates clearly that the impact of the TAE beat-driven ZS on the most unstable mode is negligible.

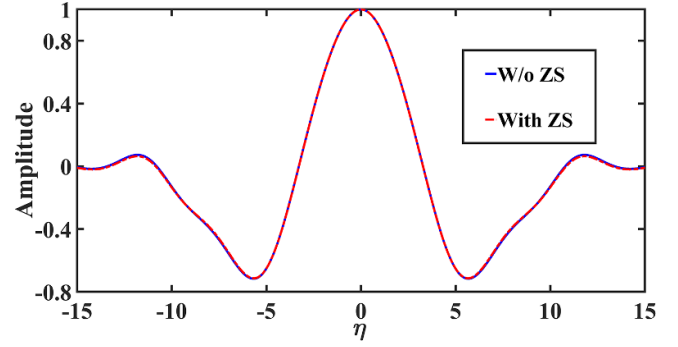


Figure 6. The mode structure of the most unstable mode. Here, $\varepsilon_{pi} = 0.06$, $q = 1$ and $b = 0.01$.

6. Summary and discussion

In this work, the indirect nonlinear interaction between TAE and ITG mode is investigated using nonlinear gyrokinetic theory and ballooning formalism, to understand the thermal plasma confinement in the presence of EPs and the associated electromagnetic oscillations. More specifically, the local stability of ITG in the presence of ZS beat-driven by finite amplitude TAE is analyzed. The governing ITG eigenmode equation in the ballooning space is derived and solved both analytically and numerically, and it is found that, for typical reactor parameters and TAE fluctuation levels, the ZS beat-driven by TAE has only weakly destabilizing effects on ITG stability in both short- and long-wavelength limits, contrary to previous speculations. This is due to the relatively weak ZS generation in the parameter region, i.e. $|e\delta\phi_Z/T_i| \sim O(10^{-4})$, partly due to the scale separation between TAE and ITG.

In the present analysis, the contributions of zonal field structure, including ZF, ZC and PSZS are accounted for on the same footing, and only the overall results are analyzed. The respective contributions of zonal field structure and PSZS to the ITG dispersion relation in the WKB limit are provided in the appendix. However, their respective contributions to the ITG stability are not analyzed, as the structure of the corresponding eigenmode equations in the ballooning space is significantly modified, due to the non-vanishing nonlinear electron response of the ITG, which makes the comparison to the present results not relevant.

The major assumptions made in the present analysis include: 1) the ZS are beat-driven by TAE, and 2) only the local stability of ITG is investigated. It is noteworthy that, in a previous study, it was found that the direct scattering of eDW by finite amplitude ambient TAE has negligible effects on eDW stability [30]. The present work, together with [30], thus, ruled out most channels for the regulation of ITG by ambient electromagnetic oscillations driven by EPs, such as TAE. Potentially effective channels to stabilize DW turbulence, including the spontaneously excited ZS by TAE via modulational instability, as well as DW radial envelope modulation by ZS, are ongoing work of the team, and will be reported in future publications.

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Appendix. Nonlinear contributions of ZFS and PSZS to ITG

Here, we will give the respective contributions of zonal field structure (ZFS, i.e. $\delta\phi_Z$ and $\delta A_{\parallel Z}$) and PSZS (i.e. δH_Z^{NL}) on the ITG linear stability. In the nonlinear gyrokinetic equation for nonlinear particle response to ITG

$$(\partial_t + v_{\parallel} \partial_I + i\omega_D) \delta H_I^{\text{NL}} = \Lambda \left[J_I \delta \phi_I (\delta H_Z^{\text{NL}} + \delta H_Z^{\text{NL}}) - J_Z \left(\delta \phi_Z - \frac{v_{\parallel}}{c} \delta A_{\parallel Z} \right) \delta H_I^{\text{NL}} \right], \quad (\text{A1})$$

the first and third term in the square bracket on the right hand side are contributions from ZFS (noting that δH_Z^{NL} is related to $\delta\phi_Z$ and $\delta A_{\parallel Z}$), while the second term corresponds to contribution from PSZS.

A.1. Nonlinear contribution of PSZS to ITG

If only PSZS contribution is kept by taking $\delta\phi_Z = 0$, $\delta A_{\parallel Z} = 0$, it results in $\delta H_Z^{\text{NL}} = 0$, the corresponding nonlinear particle response to ITG can be derived as

$$\delta H_{I,e}^{\text{NL}} = \frac{e}{T_e} F_{Me} \frac{k_{\theta I}}{k_{\theta 0}} \frac{k_{\parallel 0}}{k_{\parallel I}} \delta \hat{\phi}_0^2 \delta \phi_I, \quad (\text{A2})$$

$$\delta H_{I,i}^{\text{NL}} = -\frac{e}{T_i} |\theta_Z|^2 F_{Mi} \frac{\omega_{*i}^t}{\omega} \delta \hat{\phi}_0^2 \delta \phi_I. \quad (\text{A3})$$

Substituting equations (A2) and (A3) into the quasi-neutrality condition, the nonlinear ITG equation in the presence of PSZS beat-driven by TAE can be derived as

$$\left[\varepsilon_I - \tau \frac{\omega_{*i}}{\omega} (\chi_{iZ} + \eta_i \chi_{iZ} - 1) \delta \hat{\phi}_0^2 - \frac{k_{\theta I}}{k_{\theta 0}} \frac{k_{\parallel 0}}{k_{\parallel I}} \delta \hat{\phi}_0^2 \right] \delta \phi_I = 0, \quad (\text{A4})$$

with the term proportional to $\delta \hat{\phi}_0^2$ being the contribution of PSZS beat-driven by TAE.

A.2. Nonlinear contribution of ZFS to ITG

If we consider only the ZFS contribution by taking $\delta H_Z^{\text{NL}} = 0$, one then obtains the nonlinear particle response to ITG as

$$\delta H_{I,e}^{\text{NL}} = -\frac{e}{T_e} F_{Me} \frac{k_{\theta I}}{k_{\theta 0}} \frac{k_{\parallel 0}}{k_{\parallel I}} \delta \hat{\phi}_0^2 \delta \phi_I, \quad (\text{A5})$$

$$\delta H_{I,i}^{\text{NL}} = \frac{e}{T_i} F_{Mi} \left(|\theta_Z|^2 - 1 + \frac{\omega_{*i}^t}{\omega} \right) \frac{\omega_{*pi}}{\omega} \delta \hat{\phi}_0^2 \delta \phi_I. \quad (\text{A6})$$

Substituting equations (A5) and (A6) into the quasi-neutrality condition, the nonlinear ITG equation can be derived as

$$\left[\varepsilon_I - \tau \frac{\omega_{*i}}{\omega} \left(-\chi_{iZ} - \eta_i \chi_{iZ} + \frac{\omega_{*pi}}{\omega} \right) \delta \hat{\phi}_0^2 + \frac{k_{\theta I}}{k_{\theta 0}} \frac{k_{\parallel 0}}{k_{\parallel I}} \delta \hat{\phi}_0^2 \right] \delta \phi_I = 0. \quad (\text{A7})$$

Equation (22) can be recovered by combining the nonlinear terms of equations (A4) and (A7), i.e. the contributions of PSZS and ZFS. We however, will not compare the results of equations (A4) and (A7) to that of equation (22), as the structures of the equations are very different. More specifically, the $1/k_{\parallel I}$ terms in equations (A4) and (A7) due to non-vanishing nonlinear electron response to ITG, render the corresponding ITG eigenmode equations into a third order differential equation in ballooning space, whereas equation (22) will yield a second order differential equation in ballooning space, as the terms proportional to $1/k_{\parallel I}$ in equations (A4) and (A7) cancel each other.

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