






Chorus Wave–Driven Electron Dynamics in the Van Allen Belts: From Coherence to Diffusion

Xin Tao^{1,2} , Zeyu An^{1,2} , Fulvio Zonca^{3,4} , Liu Chen^{4,5} , and Jacob Bortnik⁶ 

¹National Key Laboratory of Deep Space Exploration/School of Earth and Space Sciences, University of Science and Technology of China, Hefei, China, ²CAS Center for Excellence in Comparative Planetology/CAS Key Laboratory of Geospace Environment, Hefei, China, ³Center for Nonlinear Plasma Science and C.R. ENEA Frascati, Frascati, Italy, ⁴Institute of Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou, China, ⁵Department of Physics and Astronomy, University of California, Irvine, CA, USA, ⁶Department of Atmospheric and Oceanic Sciences, University of California, Los Angeles, Los Angeles, CA, USA

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Key Points:

- Realistic simulations show that chorus-driven electron dynamics transition from coherent to diffusive over time
- Coherent interactions are important for wave–particle effects with timescales shorter than an electron bounce period
- Chorus-driven electron transport matches that from equivalent broadband waves and quasilinear theory over long timescales

Supporting Information:

Supporting Information may be found in the online version of this article.

Correspondence to:

X. Tao,
xtao@ustc.edu.cn

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Author Contributions:

Conceptualization: Xin Tao
Formal analysis: Xin Tao, Zeyu An, Fulvio Zonca, Liu Chen
Funding acquisition: Xin Tao
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Methodology: Xin Tao
Project administration: Xin Tao
Resources: Xin Tao
Software: Xin Tao
Supervision: Xin Tao
Validation: Xin Tao
Visualization: Xin Tao, Zeyu An
Writing – original draft: Xin Tao

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Abstract The Van Allen radiation belts contain relativistic electrons trapped by Earth's magnetic field, posing serious risks to spacecraft. Chorus waves are known to accelerate these electrons via resonant interactions, but these interactions are inherently nonlinear and coherent. How such processes shape large-scale electron dynamics remains unresolved. Two competing paradigms, nonlinear advection and diffusive transport, have been debated for decades. Here, we address this controversy using large-scale first-principles simulations that self-consistently generate realistic chorus wave fields, coupled with test particle modeling. We find that electron motion is coherent on short timescales—comparable to or less than a bounce period—but becomes stochastic over longer timescales due to decorrelation. The resulting transport coefficients support the use of quasilinear diffusion theory for long-term evolution. This work bridges microscopic nonlinear physics with macroscopic modeling frameworks, offering a unified explanation of radiation belt dynamics and advancing the foundation for space weather forecasting.

Plain Language Summary Earth's radiation belts are regions of space filled with high-energy electrons that can damage satellites and disrupt space-based technologies. One major driver of changes in these belts is a type of electromagnetic wave known as “chorus waves,” which occur naturally in space and sound like birdsong when converted to audio. Scientists have debated for years whether these waves cause electrons to change their motion gradually over time or through sudden, strong nonlinear jumps. Using advanced computer simulations that realistically generate these waves, this study shows that both behaviors can occur—but on different timescales. On short timescales, electrons can move in a rapid and coherent way, but this coherence fades quickly. Over longer times, their motion becomes random, much like particles diffusing through a fluid. This finding bridges two previously conflicting views and confirms that widely used models—based on diffusion—can still accurately describe the long-term behavior of radiation belt electrons, while also highlighting the key role of nonlinear behavior on short-timescale phenomena such as chorus wave excitation. It also provides important guidance for improving space weather forecasts that help protect satellites and other technologies operating in near-Earth space.

1. Introduction

Earth's Van Allen radiation belts were first discovered at the dawn of the space age in 1958 by the Explorer 1 and 3 spacecraft (Van Allen & Frank, 1959). More than half a century of in situ satellite observations reveal that the relativistic electron flux in the dynamic outer belt can increase by orders of magnitude during geomagnetic storms (Baker et al., 2018; Li et al., 1997; Reeves et al., 2003). The enhanced level of radiation can cause anomalies in spacecraft operations (Baker et al., 2018). Whistler-mode waves are believed to be the primary drivers of acceleration of radiation belt electrons (Chen et al., 2007; Horne et al., 2005; Reeves et al., 2013; Thorne et al., 2013). In the outer radiation belt, these waves predominantly manifest as chorus (Li et al., 2012; Tsurutani & Smith, 1974), consisting of repetitive, discrete, quasi-coherent elements (Santolík et al., 2003). Global modeling of the Van Allen radiation belt electron flux evolution often relies on quasilinear diffusion theory, which approximates chorus waves as broadband (Horne et al., 2005; Shprits et al., 2008; Thorne et al., 2013). However, this approach is controversial because chorus waves violate the basic assumptions of quasilinear theory (Albert, 2002; Allanson et al., 2024; Bortnik et al., 2008; Li & Hudson, 2019; Ripoll et al., 2020; Shi et al., 2025).

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First, chorus waves are generated by coherent nonlinear wave-particle interactions (Omura et al., 2008; Tao et al., 2021; Zonca et al., 2022), while quasilinear theory assumes wave generation by a linear instability (Kennel & Engelmann, 1966). Second, the large amplitudes and quasi-coherent nature of chorus waves lead to coherent electron motions (Albert, 2002; Bortnik et al., 2008; Omura et al., 2007), whereas quasilinear theory assumes small-amplitude, broadband waves that result in stochastic particle behavior and diffusive evolution of the phase space density. These coherent motions can result in the rapid energization of radiation belt electrons to MeV energies within seconds to minutes (Albert, 2002; Artemyev et al., 2014; Omura et al., 2007), which is not accounted for in diffusive models. Nonetheless, discrepancies exist between this nonlinear acceleration timescale and observational data, as typical variations in radiation belt electron flux during geomagnetic storms occur over hours to days (Li et al., 1997; Reeves et al., 2003). While approximating chorus waves as broadband may yield electron evolution timescales more consistent with observations (Horne et al., 2005; Thorne et al., 2013), limited satellite coverage casts doubt on whether this agreement merely reflects uncertainties in the wave parameters used in radiation belt modeling (Li & Hudson, 2019; Ripoll et al., 2020). Addressing these fundamentally opposing paradigms is critical for evaluating the validity of approximating chorus waves as broadband in radiation belt modeling and for establishing an accurate framework for predicting radiation belt dynamics, a key element of space weather forecasting.

To address the challenges posed by the quasi-coherent nature of chorus waves, previous efforts have incorporated nonlinear effects into radiation belt models by treating rapid scattering and acceleration as advection or jump processes within a transport framework (Albert, 2002; Artemyev et al., 2014). These nonlinear advection terms predict much faster electron transport than quasilinear theory, potentially leading to significant advection in phase space. However, on longer timescales, some studies show that transport remains diffusive (Artemyev et al., 2020), though with diffusion rates far exceeding quasilinear estimates, reducing the characteristic timescale to minutes (Mourenas et al., 2018). These conclusions often rely on simplified synthetic models of chorus waves that overlook key realistic properties, such as amplitude modulation, phase decoherence, and frequency fluctuation, which have been shown to be critical for correctly estimating the timescale and efficiency of nonlinear acceleration (An et al., 2022; Tao, Bortnik, Thorne, et al., 2012; Zhang et al., 2020). Satellite observations, constrained to single-point measurements, also cannot capture the full spatial and temporal evolution of chorus waves. Moreover, when repetitive chorus elements are considered, results from test particle simulations are also debated, as synthetic wave fields frequently treat these elements as overly uniform (Saito et al., 2012; Tao et al., 2014). Consequently, the long-standing debate over whether chorus-driven electron transport is dominantly advective or diffusive, and what timescales govern the resulting electron dynamics, remains unresolved.

In this work, we employ self-consistent particle-in-cell (PIC) simulations based on first principles to generate chorus wave fields with realistic features (Tao, 2014; Tao et al., 2017), overcoming the limitations of synthetic wave models used in previous test particle studies. By reproducing key observational properties of chorus waves and coupling the resulting wave fields with test particle simulations, we demonstrate that the nature of electron-chorus wave interactions depends on the timescale of interest. While interactions are coherent on short timescales comparable to the duration of a single chorus element, electron dynamics on timescales longer than a few bounce periods become equivalent to those driven by a broadband wave field.

2. Results

2.1. PIC Simulation of Chorus Waves

Previous studies have shown that PIC simulations can effectively replicate the observed characteristics of individual chorus elements (Zhang et al., 2021), including narrowband quasi-coherent structures, dynamic amplitude evolution (Tao, Bortnik, Thorne, et al., 2012), and phase decoherence (Zhang et al., 2020). Using the latest simulation techniques (Lu et al., 2021), we generate repetitive chorus elements with a typical repetition period of approximately 0.4 s, consistent with observations in the magnetosphere (Shue et al., 2015). The simulations use a background magnetic field corresponding to an equatorial radial distance of 5 Earth radii in Earth's dipole field, which is a typical outer radiation belt region, with parameters chosen to represent lower-band chorus waves. This setup ensures that the generated wave fields closely resemble those observed in situ. As shown in Figure 1b, the simulated spectrograms successfully capture the key narrowband, repetitive, and dynamic characteristics of chorus. A comparison with observational data in Figure 1a reveals similar, though not identical, elements and chirping rates. Due to computational constraints, we assume parallel propagation and neglect field line curvature

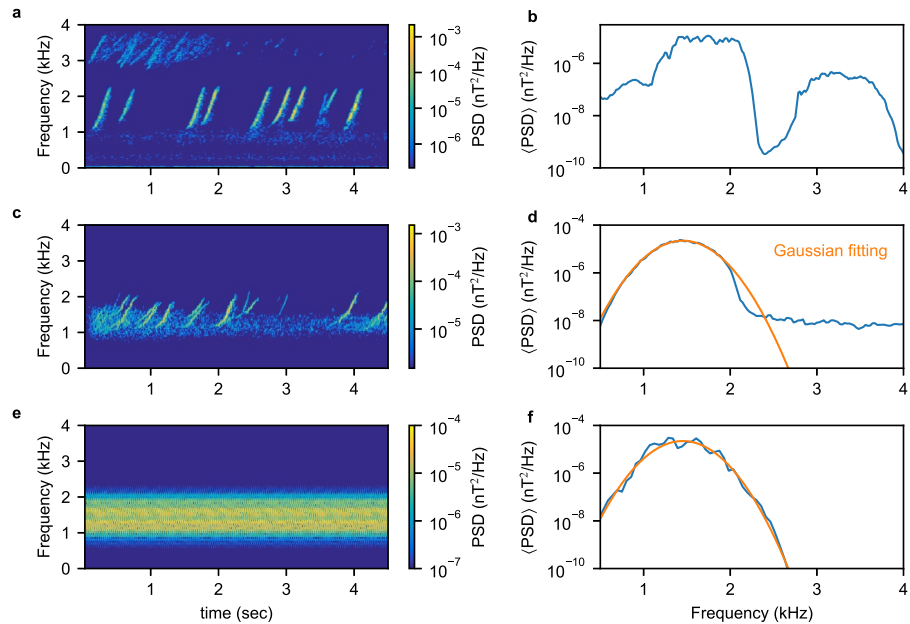


Figure 1. Typical wave fields from Van Allen Probes (Mauk et al., 2013) observations and computer simulations. (a) Whistler-mode chorus waves observed by Van Allen Probe A at $L = 5.2$ and magnetic latitude $MLAT = -1.2^\circ$ on 2014-04-30 UT, with the corresponding time-averaged power spectral density, $\langle \text{PSD} \rangle$, displayed in (b). (c) The chorus wave field from a computer simulation at an equivalent latitude of $\lambda = 3^\circ$, with the time-averaged power spectral density in (d) fitted by a Gaussian (orange line). (e) The constructed broadband wave field, whose power spectral density (f) is derived from the Gaussian fit, as indicated.

but still retain field line inhomogeneity. However, neglecting oblique propagation does not affect our main conclusions, because observed chorus waves are predominantly quasi-parallel. Furthermore, oblique waves experience Landau damping as they propagate to higher latitudes (Bortnik et al., 2006) and require larger amplitudes to initiate nonlinear interactions (Tao & Bortnik, 2010). Most importantly, if such nonlinear interactions occur, the decorrelation process described in this study applies equally well. As such, our study represents a “worst-case scenario,” where coherent, parallel-propagating waves are the most difficult to decorrelate. To assess the validity of previous global radiation belt models, we construct a broadband wave field for comparison by applying a Gaussian fit to the time-averaged power spectral density of the simulated chorus waves (Figures 1d–1f), a common approach in transport coefficient calculations (Meredith et al., 2012). Test particle simulations using both the chorus and broadband wave fields allow us to evaluate and compare their respective effects.

2.2. Test Particle Simulations

We begin by investigating whether the long-term dynamics of electrons interacting with chorus waves exhibit diffusive behavior. To this end, we perform test particle simulations using chorus wave fields generated self-consistently from PIC simulations. Figure 2 presents representative electron trajectories and the temporal evolution of their distributions for electrons with initial equatorial pitch angle $\alpha_0 = 70^\circ$ and energy $E/E_0 = 0.2$, where $E_0 = 0.511$ MeV is the electron rest energy. In Figure 2a, the equatorial pitch angle exhibits noticeable coherent variations over portions of the displayed orbits. One such trajectory, replotted in phase space, displays a characteristic circling pattern indicative of nonlinear phase trapping (Figure 2b). However, due to the discrete nature of chorus elements, the timescale of such trapping is limited. The outcomes of interactions with two consecutive chorus elements are effectively uncorrelated, as the effects of nonlinear wave–particle interactions depend sensitively on the initial interaction phase angle. In general, electrons may gain pitch angle and energy if phase-trapped or lose them if untrapped. Additionally, realistic features of chorus waves—such as amplitude modulation (Tao, Bortnik, Thorne, et al., 2012; Tao et al., 2013) and phase decoherence (Zhang et al., 2020)—captured in the PIC-generated fields enhance the stochastic nature of the dynamics and reduce the efficiency of nonlinear advective acceleration (An et al., 2022). These stochastic features are also evident in the trajectories shown in Figure 2a.

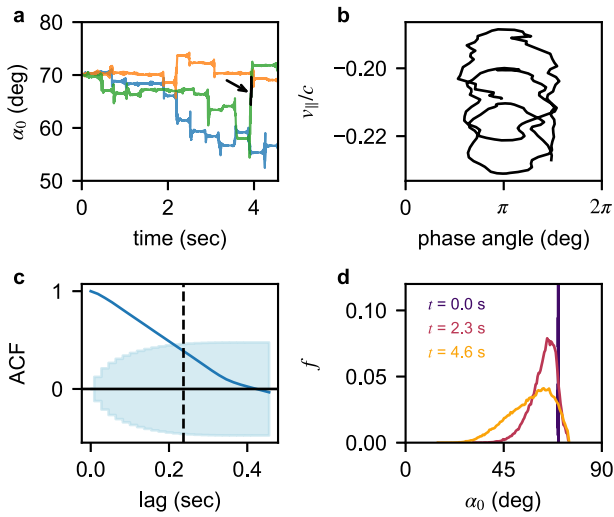


Figure 2. Time-scale dependent behavior of electrons interacting with chorus. (a) Three sample trajectories of electrons with initial energy $E/E_0 = 0.2$ and initial equatorial pitch angle $\alpha_0 = 70^\circ$. The detailed parameters used in the test particle simulations are provided in the Appendix. (b) Phase space trajectory of a phase-trapped electron, with its orbit highlighted in black and indicated by an arrow in (a). (c) Autocorrelation function of the α_0 time series for the trajectory shown in green in (a). The shaded region marks the 95% confidence interval under the null hypothesis of white noise, indicating the range within which autocorrelation values are not statistically significant. The vertical black dashed line indicates the time lag at which the autocorrelation function decays to $1/e$. (d) Evolution of the electron pitch angle distribution function $f(\alpha_0)$ at three times ($t = 0.0$ s, 2.3 s, and 4.6 s), showing progressive broadening consistent with diffusive transport.

To quantify the duration of coherence, we compute the autocorrelation function of the pitch angle time series for the trajectory shown in green in Figure 2a. As shown in Figure 2c, the autocorrelation falls below the 95% confidence bounds (shaded region) at a lag of approximately 0.2 s and remains within the bounds thereafter, indicating that pitch angle variations become statistically uncorrelated beyond this timescale. These confidence bounds represent the expected range of autocorrelation values under the null hypothesis of white noise. The decorrelation timescale—defined as the lag where the autocorrelation decays to $1/e$ —is approximately 0.24 s, or about $0.4\tau_b$, where $\tau_b = 0.61$ seconds is the bounce period for the selected energy and pitch angle. While this timescale varies among electrons, interactions are generally coherent up to approximately $\mathcal{O}(\tau_b/4)$, since chorus waves are generated near the equator and propagate toward higher latitudes, and resonant electrons must travel opposite to the wave propagation direction. Therefore, when electron trajectories are sampled at intervals significantly longer than $\tau_b/4$, their dynamics appear approximately Markovian and stochastic. Although the process is more complex, it is fundamentally similar to classical Brownian motion: a particle's motion is deterministic over short timescales (between collisions), but becomes effectively stochastic over timescales much longer than the decorrelation time (Einstein, 1905). Figure 2d shows the evolution of the pitch angle distribution function f at $t = 0.0$ s, 2.3 s, and 4.6 s. The progressive broadening of the distribution over time is indicative of diffusive rather than convective evolution. More detailed comparisons with broadband wave simulations and predictions from quasilinear theory will be presented in later sections.

2.3. Comparison of Transport Coefficients

The system's stochastic nature allows us to quantify the long-term transport of radiation belt electrons using Fokker-Planck equations, with diffusion rates and advection rates serving as key parameters. We calculate the diffusion rates, D_{EE} , and advection coefficients, A_E , for energy (E), as shown in Figure 3. These energy transport coefficients enable a direct estimation of the electron energization timescale. Other transport coefficients, such as those related to equatorial pitch angle, are directly linked to D_{EE} and A_E and are therefore not shown. We compare results for three different energies $E/E_0 = 0.2$, $E/E_0 = 0.5$, and $E/E_0 = 1.0$, representing the seed population of radiation belt electrons that can be further accelerated to relativistic energies during geomagnetic storms (Jaynes et al., 2015). Diffusion and advection coefficients are determined by linearly fitting the variance and mean of E across all test particles in the simulations. To minimize statistical noise, we use 10,000 test particles for each simulation, recording the mean and variance every $\tau_b/2$, starting from $t = \tau_b/4$. Sampling at intervals of $\tau_b/2$ ensures decorrelation of successive data points.

We compare transport coefficients for chorus waves with those for broadband waves and quasilinear theory, as shown in Figure 3. The transport coefficients for chorus and broadband waves exhibit strong agreement, except near $\alpha_0 = 90^\circ$ for $E/E_0 = 0.2$, where differences in the wave power spectral density (Figures 1d and 1f) account for the discrepancy. This agreement demonstrates that the long-term dynamics of electrons interacting with quasi-coherent chorus waves can be effectively modeled using an equivalent broadband wave field. Additionally, the comparison between quasilinear transport coefficients and those from test particle simulations shows good overall consistency, despite some differences. For instance, the maximum quasilinear diffusion coefficients exceed test particle results by a factor of 2–3, while for $E/E_0 = 0.5$ the maximum quasilinear advection coefficients are smaller by a factor of 2–4 at $\alpha_0 = 50^\circ$. These differences likely result from the simplifying assumptions of quasilinear theory, as discussed later in the context of distribution function comparisons. Overall, the transport coefficients for chorus waves, broadband waves, and quasilinear theory remain largely consistent. Consequently, our findings do not support significant secular long-term nonlinear advection arising from quasi-coherent nonlinear interactions between electrons and chorus waves, despite the existence of such advection on timescales comparable to or shorter than $\tau_b/4$.

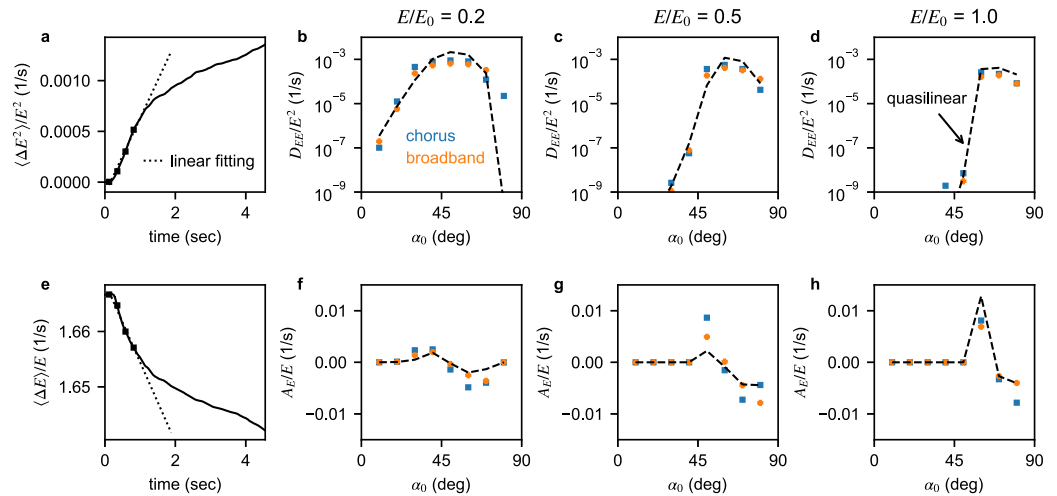


Figure 3. Comparison of transport coefficients from chorus waves, broadband waves, and quasilinear theory. (a) A sample showing variation of the variance of the energy of test particles, whose initial $\alpha_0 = 70^\circ$ and $E/E_0 = 0.5$. The test particle diffusion coefficient D_{EE} is half of the slope of the linear fitting (dotted line) to the time rate change of the variance of the energy distribution (solid line). (b) Comparison of diffusion coefficients D_{EE}/E^2 from test particle calculations using the chorus wave field (blue squares), broadband wave field (orange crosses), and quasilinear theory (black dashed lines) for the initial energy $E/E_0 = 0.2$ at eight equidistant pitch angles ranging from 10° to 80° . (c, d) Are similar to b but for initial energies $E/E_0 = 0.5$ and $E/E_0 = 1.0$, respectively. (e) A sample showing the variation of the mean of the energy, $\langle E \rangle$, of all test particles. The test particle advection coefficient is the slope of the linear fitting to the time rate change of $\langle E \rangle$. (f–h) Similar to (b–d) but comparing advection coefficients A_E/E .

2.4. Comparison of Distribution Functions

To make further comparisons between the chorus and broadband waves, we show the evolution of the distribution functions from test particle simulations for the three different energies and pitch angles in Figure 4 at $t = \tau_b/2, \tau_b, 2\tau_b$ and $4\tau_b$. At $t = \tau_b/2$, the distributions from chorus and broadband waves differ significantly due

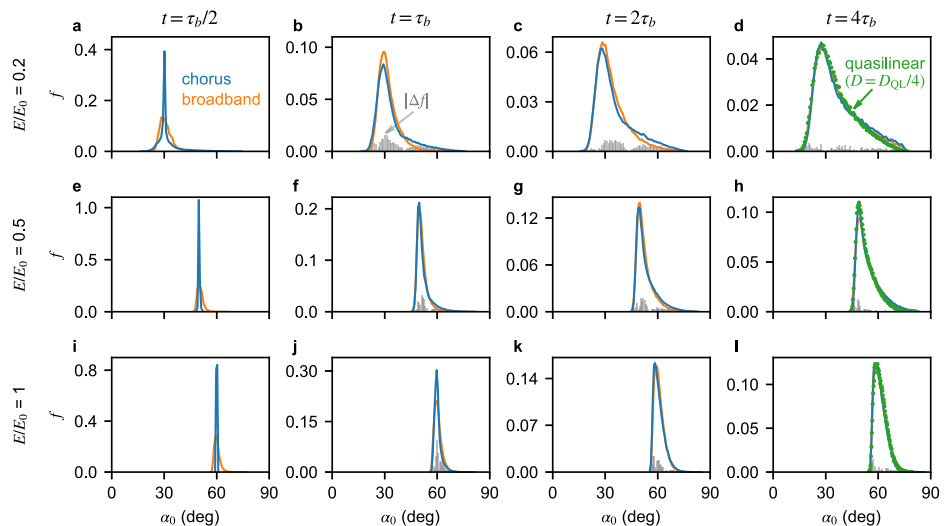


Figure 4. Comparison of electron equatorial pitch angle distributions. (a) Electron pitch angle distribution at $t = \tau_b/2$ for an initial energy of $E/E_0 = 0.2$ and an initial pitch angle $\alpha_0 = 30^\circ$. Test particle distributions from interactions with chorus waves (blue) are compared with those from broadband waves (orange). (b–d) Similar to (a) but for $t = \tau_b, 2\tau_b$, and $4\tau_b$, respectively. Gray bars indicate the absolute difference between the distributions from chorus and broadband waves, $|\Delta f|$. Green dots represent distributions predicted by quasilinear theory using $D = D_{QL}/4$. (e–h) Similar to (a–d) but for an initial energy of $E/E_0 = 0.5$ and an initial pitch angle $\alpha_0 = 50^\circ$. (i–l) Similar to (a–d) but for an initial energy of $E/E_0 = 1$ and an initial pitch angle $\alpha_0 = 60^\circ$.

to the discrete nature of the chorus elements. A substantial fraction of electrons remains unscattered by chorus, resulting in a peak near the initial pitch angle. In contrast, broadband waves continuously scatter electrons from the beginning. The discrete nature of chorus also contributes to the differences in the distribution near the initial α_0 at other times. By $t = \tau_b$ and $t = 2\tau_b$, the distributions from both wave types converge, although the distribution from chorus waves is slightly larger than that from broadband waves at pitch angles α_0 near 60° for $E/E_0 = 0.2$ and $E/E_0 = 0.5$. These differences are the effects of coherent nonlinear scattering, which can cause a large increase in electrons' pitch angle from a single interaction. At $t = 4\tau_b$, the chorus and broadband wave distributions become nearly indistinguishable. These results demonstrate again that the long-term dynamics of electrons from interactions with chorus can be well approximated by a broadband wave field due to decorrelation between successive interactions.

Figure 4 also presents comparisons of the electron distribution with predictions from quasilinear theory at $t = 4\tau_b$. We solve the Fokker-Planck equation using a Monte Carlo method based on stochastic differential equations (Tao et al., 2008). Using the theoretical diffusion coefficients (D_{QL}) results in stronger diffusion than observed in test particle simulations, as expected from comparisons of transport coefficients. Adjusting the diffusion coefficients to $D_{QL}/2$ and $D_{QL}/4$ shows that $D_{QL}/4$ achieves excellent agreement with the test particle results; therefore, only theoretical distributions using $D_{QL}/4$ are shown in the figure. Comparisons at other times are not shown for clarity of presentation, but similar agreement exists between the quasilinear and broadband wave results. It is important to note that the requirement for reduced diffusion coefficients is not due to coherent nonlinear interactions but rather to the large amplitude of the broadband waves. Quasilinear theory is designed for small amplitude waves. When the amplitude of broadband waves exceeds a certain threshold, the actual diffusion coefficients may become saturated and fall below the predictions of quasilinear theory. The exact threshold amplitude depends on the wave and particle parameters. For typical radiation belt conditions, a rough estimate from a previous study (Tao, Bortnik, Albert, & Thorne, 2012) suggests that the normalized average wave amplitude, $\delta B/B_0$ (where B_0 represents the background magnetic field), should be less than 10^{-3} for quasilinear theory to be accurate. In our simulations, the normalized average amplitude of both broadband and chorus waves exceeds 10^{-3} at latitudes greater than 5° . Correspondingly, diffusion coefficients from quasilinear theory are expected to exceed those from test particle calculations, as shown in Figure 3. A set of reduced theoretical diffusion coefficients is therefore needed to model the evolution of the distribution function, as illustrated in Figure 4. If the equivalent broadband waves had a smaller amplitude, quasilinear theory would more closely match test particle simulations for both wave types.

3. Discussion and Summary

Together, these calculations reveal the complex dynamics of electron behavior arising from interactions with chorus waves. For timescales on the order of or shorter than $\tau_b/4$, nonlinear coherent scattering by chorus significantly influences electron distributions, especially when chorus wave amplitudes are large. Such processes include chorus wave excitation and electron precipitation through scattering by individual chorus elements. However, over timescales much longer than the bounce period, the loss of coherence between successive scattering allows electron motion to be treated as stochastic. Our results indicate that, for typical chorus wave parameters in the Van Allen belts, the diffusion coefficients and long-term evolution of the electron distribution closely match those of an equivalent broadband wave field. Notably, our results do not support the presence of rapid nonlinear advection or of significantly enhanced diffusion on extended timescales. Although this study focuses on the seed population of radiation belt electrons, we expect the same decorrelation mechanism to apply to electrons from MeV to multi-MeV energies (Allison & Shprits, 2020; Hanzelka et al., 2025; Zhao et al., 2018), whose efficient acceleration generally requires lower plasma densities (Shprits et al., 2022).

It is important to note that, strictly speaking, this diffusion is not quasilinear, as chorus wave excitation involves a coherent nonlinear process. However, the quasilinear diffusion theory commonly applied in radiation belt modeling is in the resonant limit, where Van Allen belt electrons are considered to be test particles. This assumption is justified because chorus waves are mainly generated by a denser population of lower-energy keV electrons, while the relatively low density of Van Allen belt electrons (a few hundred keV to MeV) minimizes their feedback on chorus wave growth and are thus considered to be a parasitic population. The strong agreement between results for chorus and broadband waves, therefore, supports the use of quasilinear theory as a

foundational framework for radiation belt modeling in space weather prediction, provided that equivalent broadband wave amplitudes remain sufficiently small for quasilinear theory to yield accurate predictions.

Appendix A: Computer Simulation of Chorus Waves

We use a previously developed computer code, DAWN (Tao, 2014; Tao et al., 2017), which solves the Vlasov equation coupled with Maxwell's equations, to self-consistently simulate the generation of chorus waves in Earth's magnetosphere. To simulate the repetitive generation of chorus elements, we solve the following modified Vlasov equation, based on a recent study (Lu et al., 2021):

$$\frac{\partial f}{\partial t} + \dot{z} \cdot \frac{\partial f}{\partial z} + \dot{\mathbf{u}} \cdot \frac{\partial f}{\partial \mathbf{u}} = -\frac{f - f_0}{\tau}. \quad (\text{A1})$$

In this equation, f is the phase space density, z is the spatial coordinate, and $\mathbf{u} = \gamma \mathbf{v}$, where \mathbf{v} is the particle velocity and γ is the relativistic factor. The time derivative of z and \mathbf{u} are denoted by \dot{z} and $\dot{\mathbf{u}}$, respectively, with $\dot{\mathbf{u}}$ given by the Lorentz force equation. The term $-(f - f_0)/\tau$ on the right-hand side simulates the continuous injection of fresh energy into the system by restoring the total distribution to the initial state, providing free energy to generate new chorus elements. The characteristic repetition period τ is set to $25000 \Omega_{e0}^{-1}$ in normalized units, where Ω_{e0} is the equatorial electron cyclotron frequency, equivalent to approximately 0.57 s in SI units. This setting produces chorus elements with an average repetition period of about 0.4 s, consistent with typical observations in the magnetosphere (Shue et al., 2015).

In the DAWN code, the electron distribution includes two components: a cold component modeled by fluid equations and a hot component with a bi-Maxwellian distribution. The hot component has temperatures of 10.12 and 14.1 keV along and perpendicular to the magnetic field, respectively. These temperature values are chosen specifically to generate chorus elements in the lower band. The cold electron number density is chosen to satisfy $\omega_{pe}/\Omega_{e0} = 5$, where ω_{pe} is the electron plasma frequency, representing typical conditions outside the plasma-pause. The hot electron density is set at 0.6% of the cold electron density. Ions are considered fixed, as the frequency of chorus waves is much higher than the ion cyclotron frequency.

The dominant component of the background magnetic field is represented by $B_z = B_0(1 + \xi z^2)$, an approximation of Earth's dipole field without considering field line curvature. The B_x and B_y components are given by $B_x = -(x/2)dB_z/dz$ and $B_y = -(y/2)dB_z/dz$ to ensure $\nabla \cdot \mathbf{B} = 0$. Here, ξ characterizes magnetic field inhomogeneity. For a dipole field, $\xi = 4.5/(LR_E)^2$, with L being the L -shell and R_E the Earth radius (Helliwell, 1967). We set $\xi = 4.43 \times 10^{-15} \text{ m}^{-2}$ to represent Earth's dipole field at $L = 5$, which corresponds to the central region of the outer radiation belt. Our simulation domain covers both hemispheres, with the maximum latitude being about 15° , consistent with typical nightside latitudinal distributions of chorus waves from observation. Chorus waves are generated near the equator and are convectively amplified as they propagate toward higher latitudes. The normalized wave intensity, $\delta B^2/B_0^2$, is approximately 10^{-7} near the equator and reaches about 2×10^{-6} at higher latitudes. This latitude-dependent amplitude distribution is also applied in modeling the broadband wave field and in the quasilinear diffusion calculations.

Appendix B: Test Particle Simulations

Two types of wave fields are used in the test particle simulations: the self-consistent PIC-generated chorus waves and a constructed broadband wave field. The PIC-generated chorus waves are saved with a time step of $0.4\Omega_{e0}^{-1}$. For lower-band chorus waves with $\omega/\Omega_{e0} = 0.3$, this time step provides 50 data points within one wave period. The broadband wave field is constructed by superposing 100 single-frequency waves (Tao, Bortnik, Albert, & Thorne, 2012), with frequencies evenly distributed between 0 and $0.5\Omega_{e0}$. These broadband waves propagate from the equator to both hemispheres. The latitude-dependent power spectral density of the broadband waves is determined from the chorus wave field at 25 latitudes, equally spaced between the equator and the maximum latitude.

In test particle simulations, we use the same background magnetic field model as in the PIC simulation. The Boris method is used to solve the Lorentz equations of motion with a time step of $0.02\Omega_{e0}^{-1}$, ensuring accurate resolution of the electron cyclotron motion. For each simulation with a given initial energy and equatorial pitch angle, we use 10,000 test particles to reduce statistical noise. These test particles are randomly distributed in initial cyclotron and bounce phase. The diffusion coefficient D_{EE} and the advection coefficient A_E of the energy E are defined as

$$D_{EE} = \frac{\langle \Delta E^2 \rangle}{2t}, \quad A_E = \frac{\langle \Delta E \rangle}{t}. \quad (\text{B1})$$

where $\langle \Delta E^2 \rangle$ and $\langle \Delta E \rangle$ are the variance and mean of the E distribution at time t . By linearly fitting the variance and mean as functions of t , we obtain D_{EE} and A_E as

$$D_{EE} = k_D/2; \quad A_E = k_b, \quad (\text{B2})$$

where k_D and k_b are the slopes of the variance and mean fits, respectively.

Appendix C: Solving the Quasilinear Diffusion Equation

The quasilinear diffusion equation used here is a 2D bounce-averaged diffusion equation in equatorial pitch angle α_0 and momentum p ,

$$\frac{\partial f}{\partial t} = \frac{1}{G} \frac{\partial}{\partial \alpha_0} G \left(D_{\alpha_0 \alpha_0} \frac{\partial f}{\partial \alpha_0} + D_{\alpha_0 p} \frac{\partial f}{\partial p} \right) + \frac{1}{G} \frac{\partial}{\partial p} G \left(D_{\alpha_0 p} \frac{\partial f}{\partial \alpha_0} + D_{pp} \frac{\partial f}{\partial p} \right). \quad (\text{C1})$$

This equation is equivalent to the following stochastic differential equations (SDEs),

$$d\alpha_0 = A_{\alpha_0} dt + \sigma_{11} dW_1 + \sigma_{12} dW_2, \quad (\text{C2})$$

$$dp = A_p dt + \sigma_{21} dW_1 + \sigma_{22} dW_2, \quad (\text{C3})$$

where the advection coefficients, A_{α_0} and A_p , are given by

$$A_{\alpha_0} = \frac{1}{G} \frac{\partial}{\partial \alpha_0} (G D_{\alpha_0 \alpha_0}) + \frac{1}{G} \frac{\partial}{\partial p} (G D_{\alpha_0 p}), \quad (\text{C4})$$

$$A_p = \frac{1}{G} \frac{\partial}{\partial \alpha_0} (G D_{\alpha_0 p}) + \frac{1}{G} \frac{\partial}{\partial p} (G D_{pp}). \quad (\text{C5})$$

We choose $\sigma_{21} = 0$ and define the other components as (Tao et al., 2008).

$$\sigma_{11} = \sqrt{2D_{\alpha_0 \alpha_0}}, \quad (\text{C6})$$

$$\sigma_{21} = \sqrt{2D_{\alpha_0 p}^2 / D_{\alpha_0 \alpha_0}}, \quad (\text{C7})$$

$$\sigma_{22} = \sqrt{2D_{pp} - \sigma_{21}^2}. \quad (\text{C8})$$

Both dW_1 and dW_2 are increments of standard Brownian motion, calculated numerically as

$$dW(t) = \sqrt{dt} N(0, 1). \quad (\text{C9})$$

Here $N(0, 1)$ is a standard Gaussian random number with zero mean and unit variance. To solve the quasilinear diffusion equation for a given initial α_0 and E , we sample stochastic trajectories of 10,000 particles and obtain the distribution function f .

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Data Availability Statement

All data required to perform the analysis and reproduce the figures are available at Tao (2025). Due to the large size of the high-resolution PIC and test-particle simulation outputs (approximately 1.3 TB), the full data sets are not hosted online. Interested readers may contact the corresponding authors to arrange data transfer.

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References

- Albert, J. M. (2002). Nonlinear interaction of outer zone electrons with VLF waves. *Geophysical Research Letters*, 29(8), 1275. <https://doi.org/10.1029/2001GL013941>
- Allanson, O., Ma, D., Osmane, A., Albert, J. M., Bortnik, J., Watt, C. E. J., et al. (2024). The challenge to understand the zoo of particle transport regimes during resonant wave-particle interactions for given survey-mode wave spectra. *Frontiers in Astronomy and Space Sciences*, 11, 1332931. <https://doi.org/10.3389/fspas.2024.1332931>
- Allison, H. J., & Shprits, Y. Y. (2020). Local heating of radiation belt electrons to ultra-relativistic energies. *Nature Communications*, 11(1), 4533. <https://doi.org/10.1038/s41467-020-18053-z>
- An, Z., Wu, Y., & Tao, X. (2022). Electron dynamics in a chorus wave field generated from particle-in-cell simulations. *Geophysical Research Letters*, 49(3), e2022GL097778. <https://doi.org/10.1029/2022GL097778>
- Artemyev, A. V., Neishtadt, A. I., & Vasiliev, A. A. (2020). Mapping for nonlinear electron interaction with whistler-mode waves. *Physics of Plasmas*, 27(4), 042902. <https://doi.org/10.1063/1.5144477>
- Artemyev, A. V., Vasiliev, A. A., Mourenas, D., Agapitov, O. V., Krasnoselskikh, V., Boscher, D., & Rolland, G. (2014). Fast transport of resonant electrons in phase space due to nonlinear trapping by whistler waves. *Geophysical Research Letters*, 41(16), 5727–5733. <https://doi.org/10.1002/2014GL061380>
- Baker, D. N., Erickson, P. J., Fennell, J. F., Foster, J. C., Jaynes, A. N., & Verronen, P. T. (2018). Space weather effects in the Earth's radiation belts. *Space Science Reviews*, 214(1), 17. <https://doi.org/10.1007/s11214-017-0452-7>
- Bortnik, J., Inan, U. S., & Bell, T. F. (2006). Landau damping and resultant unidirectional propagation of chorus waves. *Geophysical Research Letters*, 33(3), L03102. <https://doi.org/10.1029/2005GL024553>
- Bortnik, J., Thorne, R. M., & Inan, U. S. (2008). Nonlinear interaction of energetic electrons with large amplitude chorus. *Geophysical Research Letters*, 35(21), L21102. <https://doi.org/10.1029/2008GL035500>
- Chen, Y., Reeves, G. D., & Friedel, R. H. W. (2007). The energization of relativistic electrons in the outer Van Allen radiation belt. *Nature Physics*, 3(9), 614–617. <https://doi.org/10.1038/nphys655>
- Einstein, A. (1905). On the motion of small particles suspended in liquids at rest required by the molecular-kinetic theory of heat. *Annalen der Physik*, 322(8), 549–560. <https://doi.org/10.1002/andp.19053220806>
- Hanzelka, M., Shprits, Y. Y., & Santolik, O. (2025). Nonlinear acceleration of ultrarelativistic electrons in the outer radiation belt disrupted by transverse wave modulations. *Geophysical Research Letters*, 52(18), e2025GL116883. <https://doi.org/10.1029/2025GL116883>
- Helliwell, R. A. (1967). A theory of discrete VLF emissions from the magnetosphere. *Journal of Geophysical Research*, 72(19), 4773–4790. <https://doi.org/10.1029/jz072i019p04773>
- Horne, R. B., Thorne, R. M., Shprits, Y. Y., Meredith, N. P., Glauert, S. A., Smith, A. J., et al. (2005). Wave acceleration of electrons in the Van Allen radiation belts. *Nature*, 437(7056), 227–230. <https://doi.org/10.1038/nature03939>
- Jaynes, A. N., Baker, D. N., Singer, H. J., Rodriguez, J. V., Loto'aniu, T. M., Ali, A. F., et al. (2015). Source and seed populations for relativistic electrons: Their roles in radiation belt changes. *Journal of Geophysical Research: Space Physics*, 120(9), 7240–7254. <https://doi.org/10.1002/2015JA021234>
- Kennel, C. F., & Engelmann, F. (1966). Velocity space diffusion from weak plasma turbulence in a magnetic field. *Physics of Fluids*, 9(12), 2377–2388. <https://doi.org/10.1063/1.1761629>
- Li, W., & Hudson, M. K. (2019). Earth's Van Allen radiation belts: From discovery to the Van Allen Probes era. *Journal of Geophysical Research: Space Physics*, 124(11), 8319–8351. <https://doi.org/10.1029/2018JA025940>
- Li, W., Thorne, R. M., Bortnik, J., Tao, X., & Angelopoulos, V. (2012). Characteristics of hiss-like and discrete whistler-mode emissions. *Geophysical Research Letters*, 39(18), L18106. <https://doi.org/10.1029/2012GL053206>
- Li, X., Baker, D. N., Temerin, M., Cayton, T. E., Reeves, E. G. D., Christensen, R. A., et al. (1997). Multisatellite observations of the outer zone electron variation during the November 3–4, 1993, magnetic storm. *Journal of Geophysical Research*, 102(A7), 14123–14140. <https://doi.org/10.1029/97JA01101>
- Lu, Q., Chen, L., Wang, X., Gao, X., Lin, Y., & Wang, S. (2021). Repetitive emissions of rising-tone chorus waves in the inner magnetosphere. *Geophysical Research Letters*, 48(15), e2021GL094979. <https://doi.org/10.1029/2021GL094979>
- Mauk, B. H., Fox, N. J., Kanekal, S. G., Kessel, R. L., Sibeck, D. G., & Ukhorskiy, A. (2013). Science objectives and rationale for the radiation belt storm probes mission. *Space Science Reviews*, 179(1–4), 3–27. <https://doi.org/10.1007/s11214-012-9908-y>
- Meredith, N. P., Horne, R. B., Sicard-Piet, A., Boscher, D., Yearby, K. H., Li, W., & Thorne, R. M. (2012). Global model of lower band and upper band chorus from multiple satellite observations. *Journal of Geophysical Research*, 117, A10225. <https://doi.org/10.1029/2012JA017978>
- Mourenas, D., Zhang, X.-J., Artemyev, A. V., Angelopoulos, V., Thorne, R. M., Bortnik, J., et al. (2018). Electron nonlinear resonant interaction with short and intense parallel chorus wave packets. *Journal of Geophysical Research: Space Physics*, 123(6), 4979–4999. <https://doi.org/10.1029/2018JA025417>

- Omura, Y., Furuya, N., & Summers, D. (2007). Relativistic turning acceleration of resonant electrons by coherent whistler mode waves in a dipole magnetic field. *Journal of Geophysical Research*, *112*(A6), A06236. <https://doi.org/10.1029/2006JA012243>
- Omura, Y., Katoh, Y., & Summers, D. (2008). Theory and simulation of the generation of whistler-mode chorus. *Journal of Geophysical Research*, *113*(A4), A04223. <https://doi.org/10.1029/2007JA012622>
- Reeves, G. D., McAdams, K. L., Friedel, R. H. W., & O'Brien, T. P. (2003). Acceleration and loss of relativistic electrons during geomagnetic storms. *Geophysical Research Letters*, *30*(10). <https://doi.org/10.1029/2002GL016513>
- Reeves, G. D., Spence, H. E., Henderson, M. G., Morley, S. K., Friedel, R. H. W., Funsten, H. O., et al. (2013). Electron acceleration in the heart of the Van Allen radiation belts. *Science*, *341*(6149), 991–994. <https://doi.org/10.1126/science.1237743>
- Ripoll, J. F., Claudepierre, S. G., Ukhorskiy, A. Y., Colpitts, C., Li, X., Fennell, J. F., & Crabtree, C. (2020). Particle dynamics in the Earth's radiation belts: Review of current research and open questions. *Journal of Geophysical Research: Space Physics*, *125*(5), e26735. <https://doi.org/10.1029/2019JA026735>
- Saito, S., Miyoshi, Y., & Seki, K. (2012). Relativistic electron microbursts associated with whistler chorus rising tone elements: GEMIS-RBW simulations. *Journal of Geophysical Research*, *117*(A10), A10206. <https://doi.org/10.1029/2012JA018020>
- Santolík, O., Gurnett, D. A., Pickett, J. S., Parrot, M., & Cornilleau-Wehrlin, N. (2003). Spatio-temporal structure of storm-time chorus. *Journal of Geophysical Research*, *108*(A7), 1278. <https://doi.org/10.1029/2002JA009791>
- Shi, X., Neishtadt, A. I., Artemyev, A. V., Albert, J. M., & Angelopoulos, V. (2025). Resonant wave-particle interactions near the nonlinear threshold: A type I painlevé equation based model. *Physics of Plasmas*, *32*(6), 062112. <https://doi.org/10.1063/5.0270444>
- Shprits, Y. Y., Allison, H. J., Wang, D., Drozdov, A., Szabo-Roberts, M., Zhelavskaya, I., & Vasile, R. (2022). A new population of ultra-relativistic electrons in the outer radiation zone. *Journal of Geophysical Research: Space Physics*, *127*(5), e2021JA030214. <https://doi.org/10.1029/2021JA030214>
- Shprits, Y. Y., Subbotin, D. A., Meredith, N. P., & Elkington, S. R. (2008). Review of modeling of losses and sources of relativistic electrons in the outer radiation belt II: Local acceleration and loss. *Journal of Atmospheric and Solar-Terrestrial Physics*, *70*(14), 1694–1713. <https://doi.org/10.1016/j.jastp.2008.06.014>
- Shue, J.-H., Hsieh, Y.-K., Tam, S. W. Y., Wang, K., Fu, H. S., Bortnik, J., et al. (2015). Local time distributions of repetition periods for rising tone lower band chorus waves in the magnetosphere. *Geophysical Research Letters*, *42*(20), 8294–8301. <https://doi.org/10.1002/2015GL066107>
- Tao, X. (2014). A numerical study of chorus generation and the related variation of wave intensity using the DAWN code. *Journal of Geophysical Research: Space Physics*, *119*(5), 3362–3372. <https://doi.org/10.1002/2014JA019820>
- Tao, X. (2025). Chorus wave-driven electron dynamics in the Van Allen belts: From coherence to diffusion (version 1) [Dataset]. *Zenodo*. <https://doi.org/10.5281/zenodo.17373875>
- Tao, X., & Bortnik, J. (2010). Nonlinear interactions between relativistic radiation belt electrons and oblique whistler mode waves. *Nonlinear Processes in Geophysics*, *17*(5), 599–604. <https://doi.org/10.5194/npg-17-599-2010>
- Tao, X., Bortnik, J., Albert, J. M., & Thorne, R. M. (2012). Comparison of bounce-averaged quasi-linear diffusion coefficients for parallel propagating whistler mode waves with test particle simulations. *Journal of Geophysical Research*, *117*(A10), A10205. <https://doi.org/10.1029/2012JA017931>
- Tao, X., Bortnik, J., Albert, J. M., Thorne, R. M., & Li, W. (2013). The importance of amplitude modulation in nonlinear interactions between electrons and large amplitude whistler waves. *Journal of Atmospheric and Solar-Terrestrial Physics*, *99*, 67–72. <https://doi.org/10.1016/j.jastp.2012.05.012>
- Tao, X., Bortnik, J., Albert, J. M., Thorne, R. M., & Li, W. (2014). Effects of discreteness of chorus waves on quasilinear diffusion-based modeling of energetic electron dynamics. *Journal of Geophysical Research: Space Physics*, *119*(11), 8848–8857. <https://doi.org/10.1002/2014JA020022>
- Tao, X., Bortnik, J., Thorne, R. M., Albert, J., & Li, W. (2012). Effects of amplitude modulation on nonlinear interactions between electrons and chorus waves. *Geophysical Research Letters*, *39*(6), L06102. <https://doi.org/10.1029/2012GL051202>
- Tao, X., Chan, A. A., Albert, J. M., & Miller, J. A. (2008). Stochastic modeling of multidimensional diffusion in the radiation belts. *Journal of Geophysical Research*, *113*(A7), A07212. <https://doi.org/10.1029/2007JA012985>
- Tao, X., Zonca, F., & Chen, L. (2017). Investigations of the electron phase space dynamics in triggered whistler wave emissions using low noise δf method. *Plasma Physics and Controlled Fusion*, *59*(9), 094001. <https://doi.org/10.1088/1361-6587/aa759a>
- Tao, X., Zonca, F., & Chen, L. (2021). A “trap-release-amplify” model of chorus waves. *Journal of Geophysical Research: Space Physics*, *126*(9), e2021JA029585. <https://doi.org/10.1029/2021JA029585>
- Thorne, R. M., Li, W., Ni, B., Ma, Q., Bortnik, J., Chen, L., et al. (2013). Rapid local acceleration of relativistic radiation-belt electrons by magnetospheric chorus. *Nature*, *504*(7480), 411–414. <https://doi.org/10.1038/nature12889>
- Tsurutani, B. T., & Smith, E. J. (1974). Postmidnight chorus: A substorm phenomenon. *Journal of Geophysical Research*, *79*(1), 118–127. <https://doi.org/10.1029/ja079i001p00118>
- Van Allen, J. A., & Frank, L. A. (1959). Radiation around the Earth to a radial distance of 107,400 km. *Nature*, *183*(4659), 430–434. <https://doi.org/10.1038/183430a0>
- Zhang, X.-J., Agapitov, O., Artemyev, A. V., Mourenas, D., Angelopoulos, V., Kurth, W. S., et al. (2020). Phase decoherence within intense chorus wave packets constrains the efficiency of nonlinear resonant electron acceleration. *Geophysical Research Letters*, *47*(20), e2020GL089807. <https://doi.org/10.1029/2020GL089807>
- Zhang, X.-J., Demekhov, A. G., Katoh, Y., Nunn, D., Tao, X., Mourenas, D., et al. (2021). Fine structure of chorus wave packets: Comparison between observations and wave generation models. *Journal of Geophysical Research: Space Physics*, *126*(8), e2021JA029330. <https://doi.org/10.1029/2021JA029330>
- Zhao, H., Baker, D. N., Li, X., Jaynes, A. N., & Kanekal, S. G. (2018). The acceleration of ultrarelativistic electrons during a small to moderate storm of 21 April 2017. *Geophysical Research Letters*, *45*(12), 5818–5825. <https://doi.org/10.1029/2018GL078582>
- Zonca, F., Tao, X., & Chen, L. (2022). A theoretical framework of chorus wave excitation. *Journal of Geophysical Research: Space Physics*, *127*(2), e2021JA029760. <https://doi.org/10.1029/2021JA029760>