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Semi-analytical model of ion cyclotron resonance heating antenna-plasma coupling and wave propagation in hot magnetized plasma

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Abstract

A simple model of coupling an electromagnetic wave in the Ion Cyclotron (IC) frequency domain with a plasma in tokamak configuration is presented and solved in a simplified Cartesian geometry. The electromagnetic field radiated by the antenna is accurately described by the CST Studio Suite ® that shows the electromagnetic field radiated by the antenna but does not consider the load due to the plasma just in front of the antenna itself. The Ion Cyclotron Resonance Heating (ICRH) antenna consists of several metallic straps (accommodated in a tokamak port), on which a current flows that varies sinusoidally with time. The spectrum of the electromagnetic field radiated by the antenna acts as boundary condition for the solution of the wave equation inside the plasma. A simplified equation for the IC mode inside the plasma is derived and solved in straight Cartesian geometry by considering a simplified model of the space variation of the plasma density, temperature and magnetic field profiles. In deriving the simplified wave equation, a complex dielectric tensor has been considered by separating the Hermitian (essentially given in the cold plasma approximation) from anti-Hermitian part (temperature effects and wave absorption). This model results are relevant for the Divertor Tokamak Test

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facility (DTT), a new tokamak device under construction at ENEA research center in Frascati (Italy). DTT will be equipped with an ICRH system, whose construction is ongoing.

Keywords: ICRH, ICRF, plasma, DTT

1. Introduction

Coupling electromagnetic power to a tokamak plasma via a Radio Frequency (RF) antenna for plasma heating has been a key focus of thermonuclear fusion research for several years. As far as modeling is concerned, the challenge lies in achieving both a correct representation of the propagative mode inside the plasma and a detailed geometrical description of the antenna by establishing suitable boundary conditions. Especially when the antenna frequency ranges in the domain of the ion cyclotron (IC) waves (tens to hundreds of MHz and wavelength of the order of meters), a reliable description of the electromagnetic (EM) field radiated by the antenna is fundamental for studying some physical mechanisms and designing the antenna itself [1, 2]. In general, the antenna used to couple IC waves into the plasma, consists of several adjacent straps, on which a sinusoidal current flows, and the phasing between the straps can affect the power spectrum radiated and coupled to the plasma. The straps, finite in size, are essentially oriented in the poloidal direction (the longest length) and aligned in the toroidal direction (the shortest length). In-house and commercial codes like TOPICA [3], PETRAM [4], COMSOL®, and CST® are used in order to best describe the real field radiated by the so-called strap antennas by considering the realistic toroidal geometry and the effects that the metallic wall of the tokamak can induce on the current flowing on the straps, including the loss on the coaxial transmission lines and the connection between the coaxial transmission lines and the straps. In the plasma, the situation is much more complicated, particularly in the IC domain, where the Maxwell equations describing the propagation and the absorption of the EM power must be solved in conjunction with the Vlasov equation and in a bounded inhomogeneous domain. The Maxwell-Vlasov equation system results in an integro-partial differential equation whose solution can only be obtained by numerical techniques [5–8]. Moreover, truncated to second order in Larmor radius, the system of equations is of the sixth order, indicating that inside the plasma there is a third mode (called warm or Bernstein mode and due to the finite temperature) which must be connected to the field radiated by the antenna. Unlike the antenna that couples higher frequencies like the Lower Hybrid (LH) where the warm mode can be ignored, and the cold plasma hypothesis accepted with a high degree of reliability, in the IC domain these constraints cannot be ignored. An elegant and effective procedure to cope with this problem has been adopted by M. Brambilla in the numerical codes FELICE and TORIC [9, 10], in which a consistent solution of the integro-differential equation inside the plasma with the appropriate boundary conditions has been obtained in a straight

1D Cartesian geometry (FELICE) and subsequently in a realistic 2D magnetic flux-surface geometry (TORIC). In both codes the antenna is simulated by a series of straps through which a sinusoidal current flows. The antenna description is not as accurate as in TOPICA or COMSOL, but the coupling between the antenna and the plasma and consequently the power spectra are analyzed and calculated correctly. In this paper, we present a slightly different approach compared to Koch [11], who treats damping—including the anti-Hermitian part of the dielectric tensor—directly through the Poynting theorem. In contrast, our approach aims to provide a simplified description of the electromagnetic field in the plasma, relying on the cold plasma approximation while modeling a selection of hot-plasma phenomena as perturbations. Additionally, we incorporate the anti-Hermitian term directly into the wave equation, neglecting the effect of the slow wave [12]. This approach considers only the vertical (poloidal) component of the electromagnetic field radiated by the antenna and focuses only on the fast magnetosonic mode inside the plasma. Moreover, a simple 1D geometry is adopted in this model where ‘ z ’ is the coordinate along the equilibrium magnetic field, ‘ y ’ is the vertical coordinate parallel to the strap of the antenna and ‘ x ’ is the coordinate perpendicular to the magnetic field which focuses on the inside of the plasma column. Using this approximation, the electromagnetic wave that propagates in the plasma is described by an ordinary differential equation of the second order that must be solved with the two appropriate boundary conditions. The first boundary condition is the field radiated by the antenna at the plasma edge. This field slightly depends on wave propagation itself but, when computed using electromagnetic codes with a realistic antenna geometry in front of a dielectric load that mimics plasma behavior, it well approximates the actual boundary condition under most operational conditions, even if not fully self-consistent. The second boundary condition is determined by the regularity condition, which requires the field to represent an outgoing wave far from the antenna. The analytical model presented in this work is specifically formulated to describe regions of the plasma encompassing, at most, the two cutoffs of the fast wave, while deliberately excluding the ion-ion hybrid resonance. A rigorous treatment of the ion-ion hybrid resonance would necessitate the inclusion of the Ion Bernstein branch at the mode conversion. This requirement implies that a second-order differential equation is no longer sufficient, and a fourth-order equation would be necessary to accurately represent this phenomenon. In section 2 the derivation of the 1D wave equation in the Finite Larmor Radius Approximation (FLRA) is deduced starting from the Maxwell-Vlasov equation system following the derivation recently proposed by Van Eester and Lerche in [13], and Colestock and Kashuba in [14]. In the same

section the much simpler equation (2nd order ODE) in the cold plasma limit that neglects the slow wave component of the electromagnetic field is derived. In section 3 an evaluation of the dielectric tensor elements (Hermitian and anti-Hermitian parts) included in the wave equation, is performed by using the plasma data related to the DTT tokamak, to establish the actual weight of the various elements and the role they play in the equation. In section 4 the electromagnetic calculation of the real DTT antenna is presented using the CST numerical package and the spatial distribution of the field and its Fourier transform in the parallel (to the magnetic field) direction (parallel wave-vector). In section 5 the wave equation is solved in simple configurations (constant, linear and parabolic spatial dependence of k_z^2) by imposing the appropriate boundary conditions proceeding from the CST antenna calculation, and from the plasma configuration. Finally in section 6 the conclusions are drawn. The proposed semi-analytical approach provides essential physical insights into the problem, serving as a rapid tool for identifying and analyzing key physical quantities. It precisely captures the dependencies among the physical variables involved, often overlooked in purely numerical methods.

The analysis presented in this paper primarily employs the Gaussian system for calculations and results. However, in the final section, where the absorbed power of the plasma is estimated, the results are expressed in the International System of Units (SI) to ensure consistency with global standards and facilitate broader comprehension.

2. Pertinent equations

The stationary wave equation that stems from the Maxwell-Vlasov model equation can be written as

$$\nabla \times \nabla \times \vec{E}(\vec{r}) - \frac{\omega^2}{c^2} \left(\vec{E}(\vec{r}) + \frac{4\pi i}{\omega} \vec{J}(\vec{r}) \right) = 0 \quad (2.1)$$

where $\vec{J}(\vec{r})$, the current density, is an integral relation involving the electric field itself

$$\vec{J}(\vec{r}) = \int d\vec{r}' \vec{\sigma}(\vec{r}, \vec{r}') \cdot \vec{E}(\vec{r}'). \quad (2.2)$$

In the case of a cold plasma equation (2.1) reduces to the following exact differential equation

$$\nabla \times \nabla \times \vec{E}(\vec{r}) - \frac{\omega^2}{c^2} \vec{\epsilon}^H(\vec{r}) \cdot \vec{E}(\vec{r}) = 0 \quad (2.3)$$

where the tensor $\vec{\epsilon}^H(\vec{r})$ is the Hermitian dielectric tensor of rank 2, which is defined in terms of the plasma conductivity $\vec{\sigma}(\vec{r})$ as

$$\vec{\epsilon}^H(\vec{r}) = \left(\vec{I} + \frac{4\pi i}{\omega} \vec{\sigma}^H(\vec{r}) \right). \quad (2.4)$$

Its non-zero elements are written in terms of the plasma and cyclotron frequencies and depend essentially on the plasma density and magnetic field. No thermal effects are included in the derivation (cold plasma approximation). In general, for a homogeneous, stationary and unbounded thermal plasma the dielectric tensor can be derived from the Maxwell-Vlasov system, which in this case reduces to a system of algebraic equation in Fourier space (dispersion relation; see [15]). The dielectric tensor is a complex tensor whose Hermitian component describes the propagation characteristics in the complex domain of frequency and wave-vector while its anti-Hermitian part accounts for the wave's power absorption by the plasma. Now, we consider a slab plasma in Cartesian geometry (see figure 1) where the kinetic profiles (density, temperature, magnetic field) depend only on the slab coordinate (says $\hat{x} = x/a$ along x , where ' x ' is the radial variable and ' a ' is the plasma minor radius) while, in y (vertical direction) and z (direction parallel to the external magnetic field) the plasma is homogeneous and unbounded. After Fourier transforming the wave equation equation (2.1) in y and z and expanding the integral in equation (2.2) in FLRA [13, 14] and truncating at second order, we obtain the following equation system for the electric field components $\tilde{E}_x(x, k_y, k_z)$, $\tilde{E}_y(x, k_y, k_z)$ and $\tilde{E}_z(x, k_y, k_z)$ [15, 16].

Note that in deriving equation (2.5) we have set $k_y = 0$, this is in general justified for the DTT antenna. In general, k_y is an important component of the calculation, as can be seen in [2]; For further studies considering the effects of k_y on studies related to DTT antennas, refer to [17]. The (2.5) equation system is a sixth order ordinary differential equation representing the slow mode, the fast mode and the thermal mode; the coefficients S , D and P are listed below and in the limit of cold plasma reduce to the well-known cold plasma coefficients in Stix notation [18]. The coefficients σ , δ , τ , η , ξ , π are also listed below and represent the first and second order contribution in the Larmor Radius approximation $k_x \rho_i \ll 1$ [19].

$$\begin{vmatrix} k_z^2 - \frac{\omega^2}{c^2} S - \sigma \frac{\partial^2}{\partial x^2} & - \left(i \frac{\omega^2}{c^2} D + i \delta \frac{\partial^2}{\partial x^2} \right) & ik_z (1 - \eta) \frac{\partial}{\partial x} \\ i \left(\frac{\omega^2}{c^2} D + \delta \frac{\partial^2}{\partial x^2} \right) & - \frac{\partial^2}{\partial x^2} + \left[k_z^2 - \left(\frac{\omega^2}{c^2} S - (\sigma + 2\tau) \frac{\partial^2}{\partial x^2} \right) \right] & \xi k_z \frac{\partial}{\partial x} \\ ik_z (1 - \eta) \frac{\partial}{\partial x} & - \xi k_z \frac{\partial}{\partial x} & - \frac{\partial^2}{\partial x^2} - \left[\left(\frac{\omega^2}{c^2} P - \pi \frac{\partial^2}{\partial x^2} \right) \right] \end{vmatrix} \begin{vmatrix} \tilde{E}_x \\ \tilde{E}_y \\ \tilde{E}_z \end{vmatrix} = 0 \quad (2.5)$$

Table 1. List of the main symbols used. In the following sections, the subscript α denotes the plasma species (ion or electron).

Symbol	Parameter
$\vec{E}, \vec{B}$	RF Electric and magnetic field
q_α, m_α	Electric charge and mass of plasma species α
$n_\alpha, T_\alpha, v_{th\alpha}$	Particle densities, temperatures and thermal velocities
$\omega_{p\alpha}, \Omega_{c\alpha}$	Plasma and cyclotron angular frequencies
λ_α	Larmor radius/wavelength ratio
ω	Wave angular frequency
e, c	Elementary charge, speed of the light in vacuum
$\tilde{E}_i$	Fourier transform of Electric field components $i = x, y, z$
E_i	Electric field components $i = x, y, z$
$k_x, k_z = k_\parallel$	Perpendicular and parallel wavenumbers
$n_x, n_z = n_\parallel$	Perpendicular and parallel refractive index
Ω_{Cx}^2	Complex variable of wave equation
$Z(x_{n\alpha})$	Complex plasma dispersion function

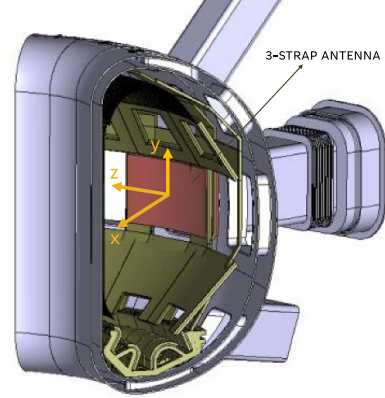
$$\begin{aligned}
S^{\text{FLR}} &= 1 - \frac{1}{2} \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} x_{0\alpha} [-Z(x_{-1\alpha}) - Z(x_{+1\alpha})] \\
D^{\text{FLR}} &= -\frac{1}{2} \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} x_{0\alpha} [Z(x_{-1\alpha}) - Z(x_{+1\alpha})] \\
P^{\text{FLR}} &= 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} x_{0\alpha}^2 Z'(x_{0\alpha}) = 1 + 2 \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega} x_{0\alpha}^2 [1 - x_{0\alpha} Z(x_{0\alpha})]
\end{aligned} \quad (2.6)$$

and

$$\begin{aligned}
\sigma &= \frac{1}{4} \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2} x_{0\alpha} \frac{v_{th\alpha}^2}{c^2} \{Z(x_{+1\alpha}) + Z(x_{-1\alpha}) - Z(x_{+2\alpha}) - Z(x_{-2\alpha})\} \\
\delta &= \frac{1}{2} \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2} \frac{v_{th\alpha}^2}{c^2} x_{0\alpha} \left\{ [Z(x_{-1\alpha}) - Z(x_{+1\alpha})] - \frac{1}{2} [Z(x_{-2\alpha}) - Z(x_{+2\alpha})] \right\} \\
\tau &= -\frac{1}{2} \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2} \frac{v_{th\alpha}^2}{c^2} x_{0\alpha} \left\{ Z(x_{0\alpha}) - \frac{1}{2} [Z(x_{-1\alpha}) + Z(x_{+1\alpha})] \right\} \\
\eta &= -\frac{1}{4} \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega \Omega_{c\alpha}} \frac{v_{th\alpha}^2}{c^2} x_{0\alpha}^2 \{Z'(x_{+1\alpha}) - Z'(x_{-1\alpha})\} \\
\xi &= -\frac{1}{2} \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega \Omega_{c\alpha}} \frac{v_{th\alpha}^2}{c^2} x_{0\alpha}^2 \left\{ Z'(x_{0\alpha}) - \frac{1}{2} [Z'(x_{+1\alpha}) + Z'(x_{-1\alpha})] \right\} \\
\pi &= \frac{1}{2} \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\Omega_{c\alpha}^2} \frac{v_{th\alpha}^2}{c^2} x_{0\alpha} \left\{ -x_{0\alpha} Z'(x_{0\alpha}) + \frac{1}{2} [x_{-1\alpha} Z'(x_{-1\alpha}) \right. \\
&\quad \left. + x_{+1\alpha} Z'(x_{+1\alpha})] \right\}
\end{aligned} \quad (2.7)$$

where $x_{n\alpha} = \frac{\omega - n\Omega_{c\alpha}}{k_z v_{th\alpha}}$ and $v_{th\alpha} = \sqrt{\frac{2\kappa T_\alpha}{m_\alpha}}$, κ is the Boltzmann constant. The complex plasma dispersion function $Z(x_{n\alpha})$ can be calculated numerically or by using the asymptotic series for large arguments [20]. Note that the coefficients in equation (2.6) are zeroth ordered, in temperature, while the coefficients in equation (2.7) are $O(\sqrt{T_\alpha})$ and $O(T_\alpha)$.

In figure 2(a) a plot of the dispersion relation (cubic in n_x^2) is shown; i.e. real part of $n_x^2 = \frac{k_x^2 c^2}{\omega^2}$ vs the slab variable obtained by equations (2.5)–(2.7) in the limit of homogeneous plasma (Fourier analysis also in perpendicular direction $\frac{\partial^2}{\partial x^2} \rightarrow -k_x^2$) for plasma parameters typical of DTT (see table 2).

**Figure 1.** Reference frame adopted for propagation model.

To validate the approximation made in obtaining figure 2(a), the Larmor-Radius-Approximation parameter $k_x \rho_i \ll 1$ is plotted in figure 2(b) vs the slab variable $\hat{x} = r/a$ for the 3 roots obtained from the solution of the cubic dispersion relation. It is possible to recognize the Fast Wave (blue line) and the thermal Ion Bernstein (black line) branches that satisfy ‘a posteriori’ to the approximation used ($k_x \rho_i \ll 1$) and the slow wave (red line). In an external layer $0.9 \leq \hat{x} \leq 1$ the FLRA seems to hold also for the ‘slow wave’ but in the outer part of the plasma $|\hat{x}| > 0.5$ the wave is evanescent ($k_x^2 < 0$) and does not appear in the real space. To extract some useful information from equation (2.5) and to show how to solve it in the plasma slab by matching the field with the opportune boundary conditions imposed by the antenna, we keep equation (2.6) in the limit of cold plasma, single propagative mode (the fast mode) and consider the damping coefficient (anti-Hermitian part of the dielectric tensor) as a perturbation of the Hermitian part. We obtain the following second order differential equation for the perpendicular component of the electric field E_y

$$\frac{\partial^2 \tilde{E}_y}{\partial x^2} - \left[\left(k_z^2 - \frac{\omega^2}{c^2} S \right) - \frac{\omega^4}{c^4} \frac{D^2}{\left(k_z^2 - \frac{\omega^2}{c^2} S \right)} - i \frac{\omega^2}{c^2} \epsilon_{yy}^A \right] \tilde{E}_y = 0. \quad (2.8)$$

Note that in the equation (2.8) S and D are the elements in Stix notation of the cold dielectric tensor [18] and the only component with a significant anti-Hermitian part is ϵ_{yy}^A that we have retained into the wave equation:

$$\epsilon_{yy}^A \approx 2\pi^{1/2} \frac{\omega_{pe}^2}{\omega^2} x_{0e} \lambda_e e^{-x_{0e}^2} + \frac{\pi^{1/2}}{2} \sum_i \frac{\omega_{pi}^2}{\omega^2} x_{0i} e^{-x_{0i}^2} \quad (2.9)$$

where $\lambda_{e/i} = \frac{k_x^2 v_{the/i}^2}{2\Omega_{ce/i}^2} = \left(\frac{2\pi \rho_{e/i}}{\lambda_x} \right)^2$.

The first term on the rhs of equation (2.9) takes into account the damping of electron species when $x_{0e} = \frac{\omega}{k_z v_{the}} = O(1)$ i.e. $v_{zphase} = \frac{\omega}{k_z} \approx v_{the}$ while it is negligible for ions because $x_{0i} = \frac{\omega}{k_z v_{thi}} \gg 1$. The second term on the rhs accounts for the ion damping at the fundamental harmonic resonance of the minority ions when $x_{1i} = \frac{\omega - \Omega_{ci}}{k_z v_{thi}} \approx 0$, i.e. $\omega \sim \Omega_{ci}$, and it is

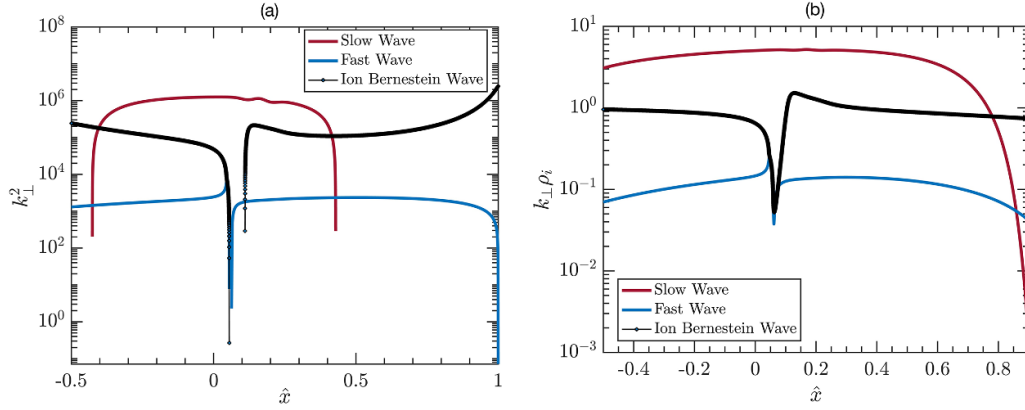


Figure 2. (a) Solution of the Dispersion Relation in FLRA showing $k_{\perp}^2$ vs the slab variable $\hat{x} = r/a$ (perpendicular to the magnetic field direction z) for plasma parameters typical of DTT in minority heating scheme $n_z = \frac{ck_z}{\omega} = 5$, ^3He minority (5%), see parameters table 1 below. Note that $\hat{x} = 1$ ($r = a$) corresponds to the plasma edge, $\hat{x} = 0$ is the plasma center, while negative values of x represent the high field side of the plasma column. (b) Plot of FLRA parameter vs the slab variable $\hat{x} = r/a$ (perpendicular to the magnetic field direction z) for plasma parameters typical of DTT in minority heating scheme for the same plasma parameters of figure 2(a).

negligible for electrons because in this case we always have $\omega \ll \Omega_{ce}$. A third term can also be considered that accounts for the ion damping at the first harmonic of the majority species if present within the plasma (for example in the case of Helium 3 minority in Deuterium majority plasma).

The scenario described through equation (2.9) holds in the case of minority heating scheme that is the most common scheme to heat the plasma ions in DTT tokamak. The anti-Hermitian part of the ‘yy’ tensor element can be written for electron and ion (Deuterium and Helium 3) damping as follows

$$\epsilon_{yy}^A \approx \pi^{1/2} \left[\frac{k_x^2 \omega_{pe}^2}{k_z^2 \Omega_{ce}^2} \frac{1}{x_{0e}} e^{-x_{0e}^2} + \frac{1}{2} \frac{\omega_{pD}^2}{\omega^2} x_{0D} e^{-x_{0D}^2} + \frac{1}{2} \frac{\omega_{pHe3}^2}{\omega^2} x_{0He3} e^{-x_{0He3}^2} \right]. \quad (2.10)$$

The homogeneous limit of equation (2.8) is essentially the dispersion relation whose real part can be accounted as

$$k_x^2 = - \left(k_z^2 - \frac{\omega^2}{c^2} S \right) + \frac{\omega^4}{c^4} \frac{D^2}{\left(k_z^2 - \frac{\omega^2}{c^2} S \right)} \quad (2.11)$$

that describes the fast branch of the dispersion diagram (blue curve of the plot of figure 2(a)). Indeed summarizing we have to solve the following second order ordinary differential equation for the y component (perpendicular to both the magnetic field direction z and the slab variable x) of the electric field:

$$\frac{\partial^2 \tilde{E}_y(k_z, x)}{\partial x^2} + \Omega_{Cx}^2(k_z, x) \tilde{E}_y(k_z, x) = 0 \quad (2.12)$$

where

$$\Omega_{Cx}^2(k_z, x) = k_x^2(k_z, x) [1 + i\gamma(k_z, x)]$$

or by using the wavenumbers n_x , and n_z

$$\Omega_{Cx}^2(n_z, x) = \frac{\omega^2}{c^2} n_x^2(n_z, x) [1 + i\gamma(n_z, x)] \quad (2.13)$$

and

$$\gamma(n_z, x) = \sqrt{\pi} \left(\frac{\omega_{pe}^2}{\Omega_{ce}^2} \frac{e^{-x_{0e}^2}}{n_z^2 |x_{0e}|} + \frac{1}{2} \frac{\omega_{pD}^2}{\omega^2} \frac{|x_{0D}|}{n_x^2} e^{-x_{0D}^2} + \frac{1}{2} \frac{\omega_{pHe3}^2}{\omega^2} \frac{|x_{0He3}|}{n_x^2} e^{-x_{0He3}^2} \right). \quad (2.14)$$

Note that $\gamma(n_z, x)$ is positive definite, and the coefficient $\Omega_{Complex}^2(n_z, x)$, can be positive or negative depending on the local values of the quantities S , D and n_z . Figure 3(a) shows a plot of the coefficient $\sqrt{\Re(\Omega_{Cx}^2(n_z, x))}$ vs n_z at fixed radial position (density and magnetic field kept constant, and equal to the edge values: $n_e = 3.8 \times 10^{12} \text{ cm}^{-3}$, and $B_{\text{edge}} = 4.7 \text{ T}$) where it is possible to notice that in correspondence of the sign inversion of the coefficient $\Re(\Omega_{Cx}^2(n_z, x))$ we have propagative $\Re(\Omega)$ or evanescent $\Im(\Omega)$ wave; the plasma parameters are the same of figure 2. In figure 3(b) the real part of the function $\sqrt{\Re(\Omega_{Cx}^2(n_z, x))}$ is plotted vs parallel wavenumber n_z for various plasma densities from 10^{11} cm^{-3} up to 10^{13} cm^{-3} for which the coefficient is positive and the wave is propagative. The plot essentially tells us that to couple (without evanescence) the part of the spectrum that lies beyond the value $n_z = 8$, in front of the antenna the density must be greater than 10^{13} cm^{-3} .

In section 4 the solution of the above equation is obtained analytically when the plasma parameters are kept constant along the slab variable or when the coefficient $\Omega_{Cx}^2(n_z, x)$ is varying linearly with the slab variable x , or $\Omega_{Cx}^2(n_z, x)$ behaves like a parabola. The arbitrary constants are determined by choosing the appropriate boundary conditions at the antenna-plasma interface.

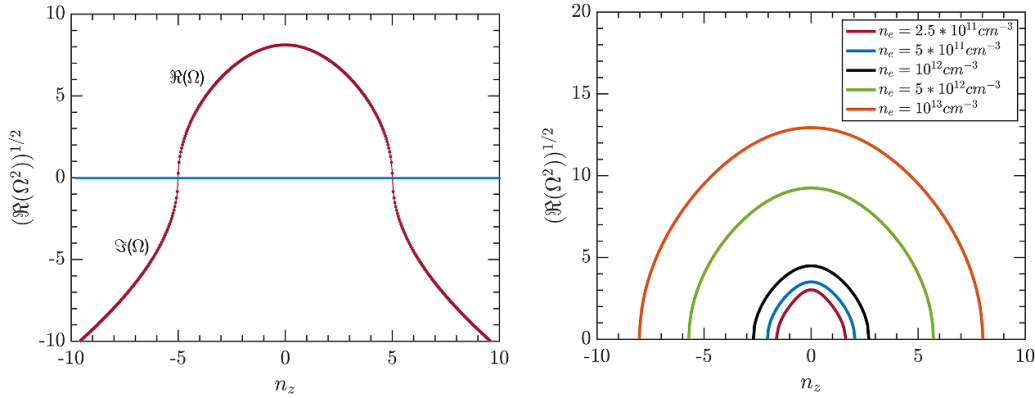


Figure 3. (a) Figure 4 Plot of the coefficient $\sqrt{\Re(\Omega_{Cx}^2)}$ vs n_z at fixed radial position (density and magnetic field kept constant, and equal to the edge values: $n_{\text{edge}} = 3.8 \times 10^{12} \text{ cm}^{-3}$, $B_{\text{edge}} = 4.7 \text{ T}$, $T_e = T_i = 130 \text{ eV}$). (b) Figure 4 Plot of the coefficient $\sqrt{\Re(\Omega_{Cx}^2)}$ vs n_z for several values of densities in front of the antenna.

3. Evaluation of the full complex dielectric tensor elements and its comparison with the FLRA and ‘cold wave limit’ expression

One of the aims of this work concerns the study of the plasma dielectric tensor that describes the response of a plasma to electromagnetic waves in magnetized plasma. This section will present the parametric studies of the dielectric tensor components implemented in MATLAB, conducted for the evaluation of three different forms of the dielectric tensor: COLD approximation, FLR approximation, and FULL complete expression. These parametric studies involve the tensor elements relevant to the 1D propagation model. This section presents the analytical forms of the dielectric tensor that have been examined in parametric studies. The non-local nature of the plasma constitutive relation is due to the thermal spread of the particle velocities; it disappears if the temperature tends to zero or if the temperature effects have a negligible influence on the wave propagation. The limit form of $\bar{\epsilon}$ when spatial dispersion is neglected is called the cold plasma dielectric tensor. The cold plasma dielectric tensor remains meaningful also in spatially non-homogeneous plasmas. Thus, in the cold limit the electrodynamics of plasmas is like that of ordinary dielectrics, except for the anisotropy due to the static magnetic field and the resulting optical activity. The dielectric tensor of a cold plasma is easily obtained from general form (see [19]) by putting the temperature T_α of all species equal to zero. Adopting the convenient notation of Stix, the components that remain finite in this limit are S and D (also P is finite, but it is not relevant for the approximation made subsequently). To investigate thermal effects on the dispersion relation, it is useful to analyze an expansion of the hot-plasma dielectric tensor to the first order in the temperature [19]. To perform this, it is useful to recall the different character of the motion of charged particles parallel and perpendicular to a uniform static magnetic field. In the direction perpendicular to B_0 a particle always remains within one Larmor radius from the line of force on which its guiding center is moving. This limits the range of spatial dispersion in this direction

and provides a natural small parameter for the expansion of the dielectric tensor, i.e. the ratio of the Larmor radius of a thermal particle to the perpendicular wavelength $\rho_\alpha/\lambda_x \ll 1$. In the dielectric tensor this ratio enters through the dimensionless quantity $\lambda_\alpha = \left(\frac{2\pi\rho_\alpha}{\lambda_x}\right)^2$ in the modified Bessel function argument. Along B_0 , particles move freely with constant velocity. Hence, we cannot perform an expansion of the dielectric tensor with respect to parallel direction. The FLRA is obtained by developing the dielectric tensor to first order (i.e. in temperature) for each species of charged particles. Parallel dispersion is also weak if for all species the additional conditions $\frac{\omega - \Omega_{ca}}{k_z v_{\text{tha}}} \gg 1$ is satisfied. This condition is in general satisfied far from the layer where the cyclotron resonance $\omega = n\Omega_{ca}$ occurs, i.e.: $\frac{2\pi\rho_\alpha}{\lambda_x} \ll \frac{\delta\omega}{\Omega_{ca}}$. From this condition, a fully algebraic dispersion relation can be obtained by expanding the Z functions for large argument and keeping the first two terms up to T_α . Because of the asymmetry between the plasma response in the parallel and perpendicular directions, the FLR expansion naturally leads us to write the dispersion relation as a polynomial as established in equations (2.5)–(2.7). In general, the FLRA connected to the weak spatial dispersion in parallel direction similarly to the cold limit makes the dielectric tensor Hermitian. The anti-Hermitian part in fact, also in the case of finite temperature and far from the resonance (see conditions above), can be neglected because the temperature appears in the anti-Hermitian part as an argument of a decaying exponential $\approx (e^{-\left(\frac{\omega - \Omega_{ca}}{k_z v_{\text{tha}}}\right)^2})$. In our case, when obtaining the wave equation equation (2.8) above, we have considered the anti-Hermitian part of the dielectric tensor element ϵ_{yy} as a perturbation of the Hermitian part, while it has been assumed that the anti-Hermitian part of ϵ_{xx} and ϵ_{xy} can be totally neglected. In what follows we perform a parametric study of the Hermitian and anti-Hermitian part of the above quoted dielectric tensor elements as function essentially of the temperature, parallel wave number and magnetic field to establish the validity of the FLRA for both Hermitian and anti-Hermitian part of the dielectric tensor elements. This study allows us to understand the choice we have done in describing the wave damping as a

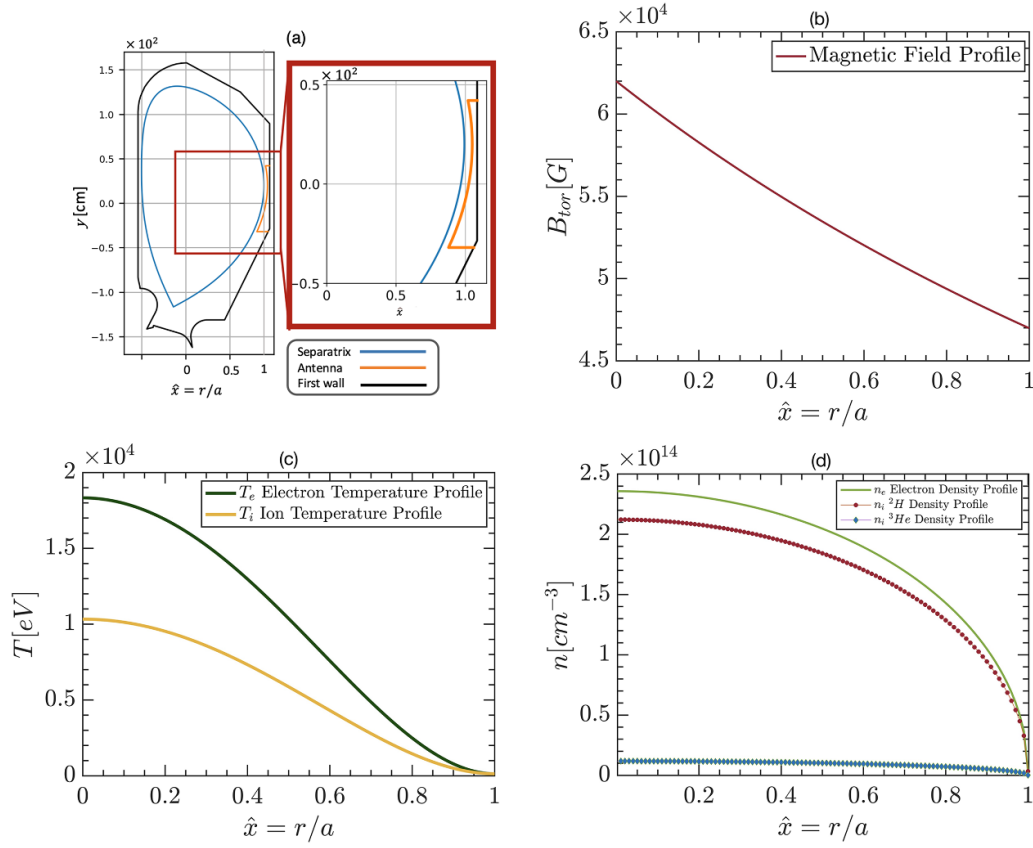


Figure 4. (a) Reference frame of the DTT minority heating scenario. Antenna placed at $\hat{x} = 1$ and plasma center at $\hat{x} = 0$. (b) Magnetic field profile $B_{tor}(\hat{x})$. (c) Electron temperature profile $T_e(\hat{x})$ and Ion temperature profile $T_i(\hat{x})$. (d) Electron density profile $n_e(\hat{x})$ and ion density profile $n_i(\hat{x})$.

perturbation in equation (2.8) essentially only due to the ‘yy’-component (anti-Hermitian part) of the dielectric tensor.

The calculated elements have been employed in a DTT minority heating scenario constructed from the integrated modeling of DTT reference scenarios [21]. The reference system associates the ‘z’ axis with the toroidal direction, the ‘y’ axis with the poloidal direction, and the ‘x’ axis with the radial direction normalized to the plasma radius a (see figure 4(a)).

We have chosen to analyze a Deuterium (^2H) plasma with a minority ^3He (at 5%) with the following characteristics:

- magnetic field profile $B(\hat{x}) = \frac{B_0}{(1+a\hat{x}/R_0)}$ (figure 4(b)) (table 2);
- electron and ion temperature profiles

$$T_{e/i}(\hat{x}) = T_{0_{e/i}}(1 - \hat{x}^{\alpha_{T}})^{\beta_T} + T_{edge} \text{ (figure 4(c)) (table 2);}$$

- electron and ion density profiles

$$n_{e/i}(\hat{x}) = n_{0_{e/i}}(1 - \hat{x}^{\alpha_n})^{\beta_n} + n_{edge} \text{ (figure 4(d)) (table 2);}$$

- frequency of the radiating antenna $f = 60$ MHz.

The choice and value of these parameters come from previous studies and characterization, done in the framework of DTT

Table 2. Main Parameters of DTT Minority Heating Scenario. The plasma radius a is intended from the center of the plasma to the edge.

Parameter	Symbol	Value
Major Radius	R_0	219 cm
Plasma radius	a	72.7 cm
Toroidal Field	B_0	6.2×10^4 G
Central electron temperature	T_{0_e}	18.2 keV
Central ion temperature	T_{0_i}	10.2 keV
Edge temperature	T_{edge}	130 eV
Central electron density	n_{0_e}	$2.32 \times 10^{14} \text{ cm}^{-3}$
Edge density	n_{edge}	$3.8 \times 10^{12} \text{ cm}^{-3}$
α temperature parameter	α_T	2
β temperature parameter	β_T	2.5
α density parameter	α_n	2
β density parameter	β_n	0.5

project [21], which will not be discussed here. In particular, the magnetic field on axis is 6.2 T at $\hat{x} = 0$ [22]. The density and temperature profiles were modeled based on typical profiles found in [22]. The antenna frequency is within the operating range of the ICRH for DTT [23]. Finally, the n_z range was derived from previous studies related to the radiated power spectrum of the 3-strap antenna [24]. It is worth to note that all profiles and the subsequent parametric studies refer to a

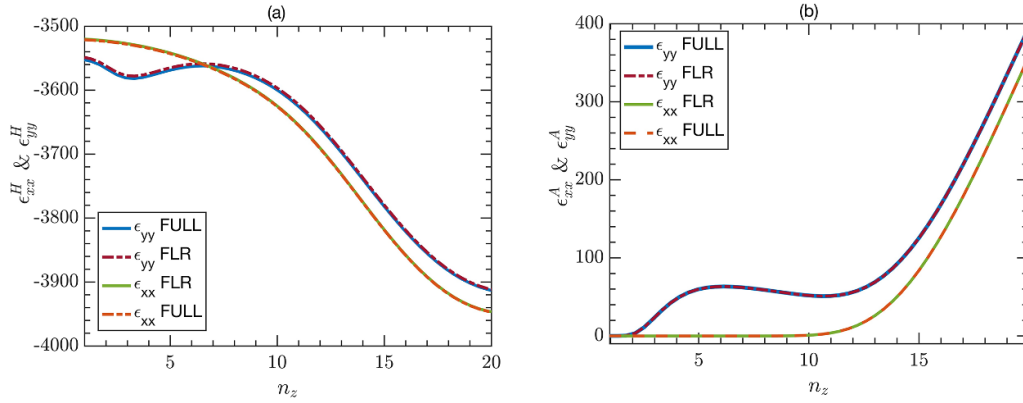


Figure 5. Hermitian (a) and anti-Hermitian (b) part of the tensor elements ϵ_{xx} and ϵ_{yy} vs n_z at fixed $\hat{x}$ with local plasma parameters $n_e = 2.12 \times 10^{14} \text{ cm}^{-3}$, $T_e = 12.56 \text{ keV}$, $T_i = 7.58 \text{ keV}$ and $B = 5.5 \text{ T}$.

configuration in which the antenna is placed at $\hat{x} = 1$, moving towards $\hat{x} = 0$ means we are going towards the center of the tokamak (see figure 4(a)).

This study focuses on three tensor elements ϵ_{xx} , ϵ_{xy} and ϵ_{yy} in a one-dimensional propagation model, conducting a parametric analysis of their behavior. The Hermitian and anti-Hermitian parts of ϵ_{xx} were evaluated at plasma conditions $n_e = 2.12 \times 10^{14} \text{ cm}^{-3}$, $T_e = 12.56 \text{ keV}$, $T_i = 7.58 \text{ keV}$. The study found that the effects of temperature and parallel wavenumber on the Hermitian part of ϵ_{xx} were minimal, with a maximum relative error of about 7% for $n_z = 20$. The cold plasma approximation is acceptable, and the FLRA and full calculations were in excellent agreement. For the anti-Hermitian part, the FULL and FLRA models were almost zero and closely aligned up to $n_z \approx 12$, indicating significant temperature and parallel wavenumber effects only for $n_z > 12$. The contribution to the wave absorption in any case remains so weak (as will be shown later) that it is justified to neglect it in the wave equation.

Similar analysis for ϵ_{yy} revealed a slight difference in the Hermitian part between the full calculation and FLRA, likely due to neglecting the first harmonic resonance in the approximate formula. Nonetheless, the cold plasma approximation remained a good fit. The anti-Hermitian part showed consistent behavior between the full and FLRA models, diverging from the cold limit over a wide range of n_z .

In figure 5 the comparison between the Hermitian (5(a)) and anti-Hermitian (5(b)) part of ϵ_{xx} and ϵ_{yy} is depicted vs the parallel wavenumber and for $\hat{x} = 0.4$.

Although in the Hermitian case the parallel wavenumber has almost no influence on the trend of ϵ_{xx} in comparison with ϵ_{yy} , vice versa in the anti-Hermitian case, the parallel wavenumber and electron temperature affect strongly the behavior of ϵ_{yy} with respect to ϵ_{xx} , showing a relevant increase also for low parallel wavenumbers in the range $2 \leq n_z \leq 15$. At higher n_z the value of the anti-Hermitian ϵ_{xx} and ϵ_{yy} tends to increase without any significant difference. It is important to stress here that this parametric study is relevant in the scheme

of power absorption of the electron species by Transit Time Magnetic Pumping (TTMP) damping [25]. The ion dynamics is unaffected by this study and in any case for ICRH waves the ion dynamics is involved only in the proximity of the resonance $\omega - n\Omega_{ci} \pm k_{\parallel} v_{thi} = 0$. To this end we have plotted in figure 6(a) the Hermitian part of ϵ_{xx} and ϵ_{yy} , ϵ_{xy} in FLRA and cold plasma vs $\hat{x}$, for plasma parameters as in table 2. It is worth noting that in the case of cold plasma the IC resonance is visible and in the ‘FLRA’ its divergence is removed by temperature effects.

In figure 6(b) the anti-Hermitian part of the above dielectric elements ϵ_{xx} , ϵ_{yy} , ϵ_{xy} is depicted, showing that in the outer part of the plasma far from the IC resonance the elements ‘xx’ and ‘xy’ are negligible (less than 10^{-5}), while near the resonance they increase a lot due to the IC resonance. Far from the resonance only the element ‘yy’ shows a finite value due to the electron TTMP damping.

4. Evaluation of the antenna radiated field for DTT ICRH

The ICRH inside the DTT experiment involves the use of an antenna system, which is responsible for launching radio waves into the plasma to heat it and control its properties. The antenna system consists of a set of antennas that are positioned around the tokamak vessel. The antennas are designed to launch radio waves within a specific frequency range into the plasma [21]. The frequency of the waves is carefully chosen to match the cyclotron frequency of the ions in the plasma, allowing them to efficiently absorb the energy from the waves and heat up. In the DTT facility the ICRH system shall couple 6 MW of power to the plasma in the 60–90 MHz frequency range by means of four launchers. The launching structure has been studied and optimized by using the commercial RF simulation software CST Studio Suite. In those simulations plasma is considered as an equivalent, high permittivity, lossy dielectric and vacuum layers [26]. The three straps antenna modeled in CST is shown in figure 7.

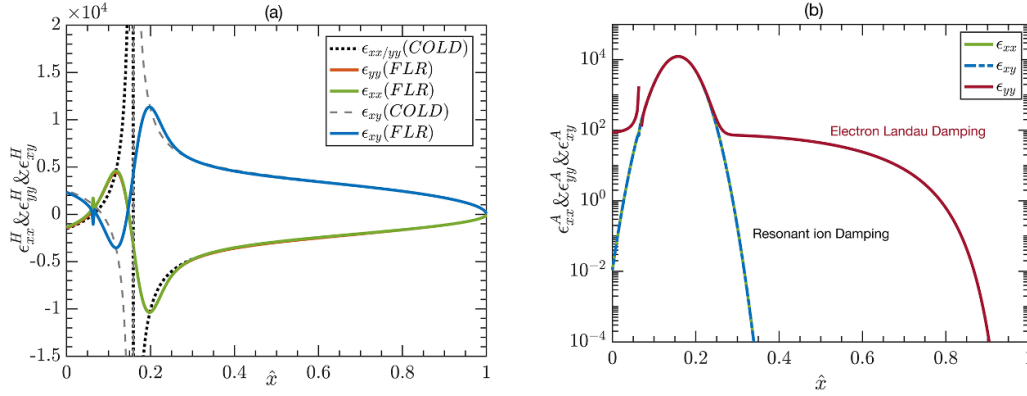


Figure 6. Evolution of the Hermitian (a) and anti-Hermitian (b) part of the dielectric tensor elements, ϵ_{xx} , ϵ_{yy} , ϵ_{xy} vs $\hat{x}$ for the plasma parameters as in table 2.

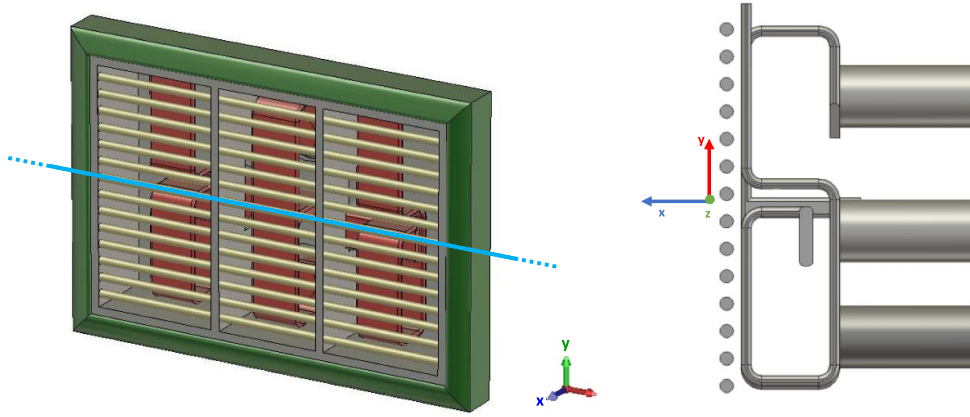


Figure 7. (a) Flat geometry of ICRH 3-strap antenna for DTT with the extraction line of the field component E_y . (b) Side view of the antenna and the extraction line of the field E_y .

Through CST, we conducted an evaluation of the fields radiated by the 3-strap antenna, and finally, we performed an analysis of the k -spectra. Field extraction has been carried out by considering a simplified geometry of the 3-strap antenna, specifically a flat geometry that has been used as a first approximation of the actual curved geometry.

The electric field has been extracted, at the frequency of 60 MHz, 0.9 cm far from the Faraday Screen of the antenna, towards the inner part of the Tokamak, along specific directions useful for subsequent 1D analysis of wave propagation. In particular, the extracted field component is $E_y(y, z)$. Fourier transform has been performed on this field following the well-known theoretical formulation:

$$\tilde{F}(n_z) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} f(z) e^{ik_0 n_z z} dz. \quad (4.1)$$

As an example, figure 8 reports the real and imaginary parts of the $E_y(y, z)$ component at $y = 0$ and the corresponding Fourier transform $\tilde{E}_y(y = 0, n_z)$.

5. Solution of the wave equation with the boundary conditions at the IC antenna

The easiest manner to couple the IC antenna radiated field in vacuum to the plasma is to consider the solution of equation (2.12) and to match it at the boundary (plasma-vacuum interface) with the field radiated by the antenna: i.e. $\tilde{E}_y^{\text{plasma}}(n_z, \hat{x} = 1) = \tilde{E}_y^{\text{antenna}}(n_z, \hat{x} = 1)$, where $\tilde{E}_y^{\text{antenna}}$ is the field spectrum. Before solving equation (2.12) by keeping into account the real space dependence of the Ω_{Cx}^2 (the real and the imaginary parts given by equations (2.13) and (2.14), respectively), we will give some analytical solutions of equation (2.12), as a guideline, for an analytical dependence of the Ω_{Cx}^2 over the slab variable and n_z .

In figures 9(a) and (b) the real and the imaginary part of Ω_{Cx}^2 are plotted for the plasma parameters as given in table 2 and related profiles of density, temperatures and magnetic field, and for several n_z . As it is possible to see in the plot the real part of Ω_{Cx}^2 shows a cut-off at low density for certain values of n_z . The n_z -interval of the cut-off changes with space (see also plots in figure 3). Moreover, a cut-off at higher density, close to the two-ion hybrid resonance, occurs near the plasma center. Conversely in figures 9(c) and (d), the variation of Ω_{Cx}^2

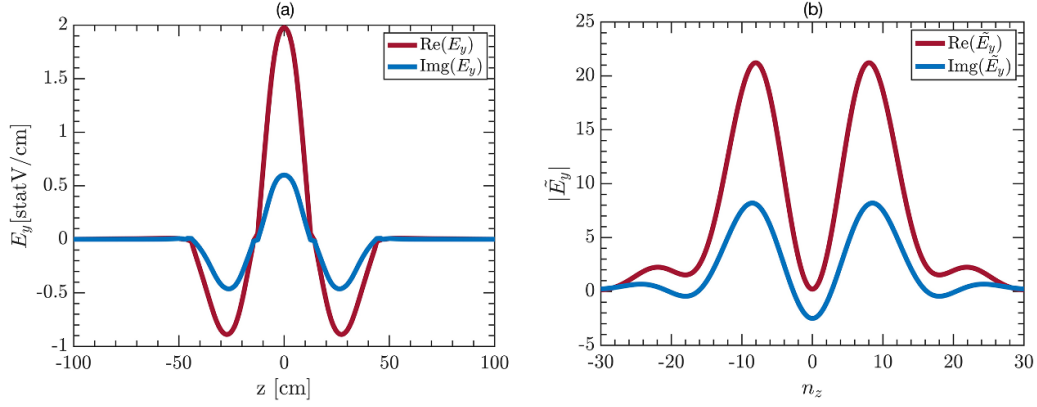


Figure 8. Real part of $E_y(y=0, z)$ and imaginary part of $E_y(y=0, z)$ (a) and the Fourier transform (b).

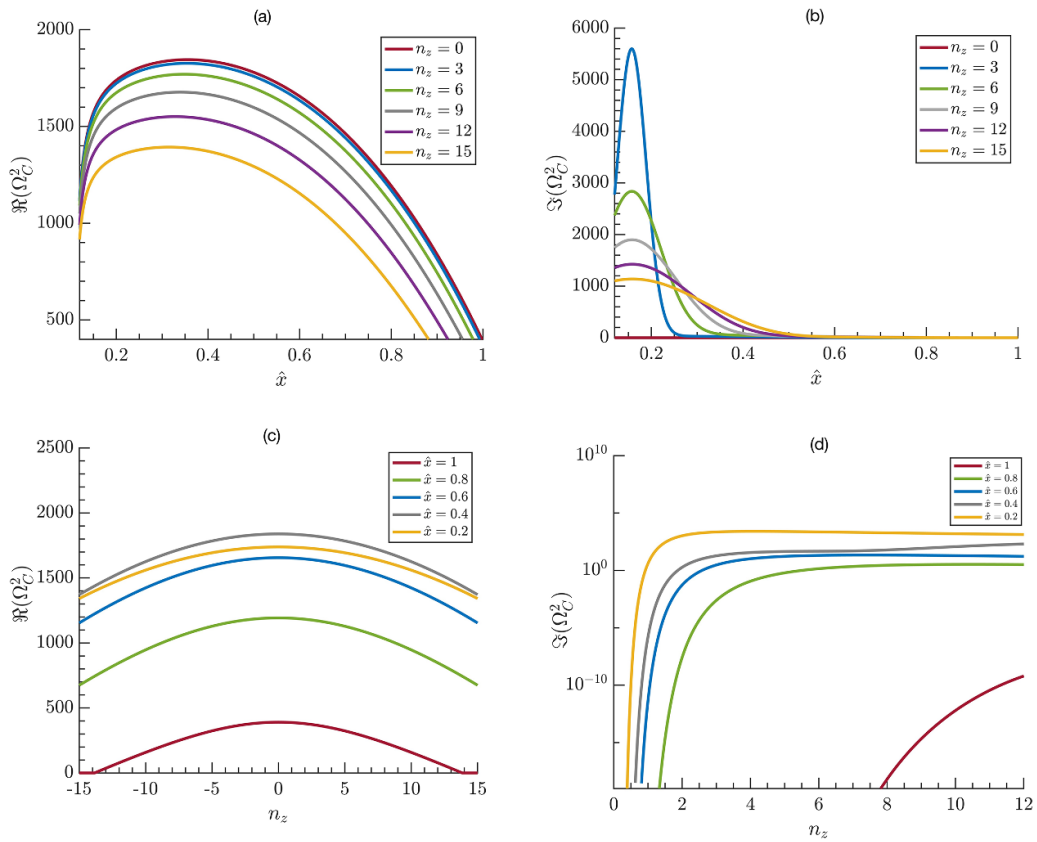


Figure 9. Real (a) and Imaginary (b) part of $\Omega_{Cx}^2(n_z, \hat{x})$ vs $\hat{x}$ for several n_z and plasma parameter as in table 2. Real (c) and Imaginary (d) part of $\Omega_{Cx}^2(n_z, \hat{x})$ vs n_z for several slab positions and plasma parameter as in table 2.

as function of n_z , is shown at 5 radial points. Figure 9(c) shows that the variation of the real part of Ω_{Cx}^2 is very weak and in our calculation, it can be neglected. Figure 9(d) shows the variation of the imaginary part of Ω_{Cx}^2 for the same 5 radial points, and it tends to increase by increasing n_z , with a behavior that follows an exponential fit.

5.1. Homogeneous case

As a simple exercise that has methodological value, we start solving equations (2.12) and (2.13) when $\Omega_{\text{Complex}}^2(n_z, \hat{x})$ is

constant in space but can vary as function of n_z . However, as shown in figures 9(a) and (b) where we have plotted the real behavior of $\Omega_{\text{Complex}}^2(n_z, \hat{x})$, this assumption appears quite simplified. Nonetheless, this simple exercise proves very useful for understanding the solution approach to equations (2.12) and (2.13).

If we assume both the real and imaginary parts of Ω_{Cx}^2 are constant and perform the coordinate change $\xi = (1 - x/a)$, then we can write (2.12) as follows:

$$\frac{\partial^2 \tilde{E}_y(\xi, n_z)}{\partial \xi^2} + a^2 \Omega_{Cx}^2(n_z) \tilde{E}_y(n_z) = 0. \quad (5.1)$$

While redefining $\Omega_H(n_z) = a\Omega_{Cx}(n_z, \hat{x})$, so the solution of the differential equation (2.12) is

$$\tilde{E}_y(\xi, n_z) = C_1 e^{i\Omega_H(n_z)\xi} + C_2 e^{-i\Omega_H(n_z)\xi}. \quad (5.2)$$

Far from the IC resonance in the zone where $\gamma(n_z) \ll 1$, (see figures 9(a) and (b)) the complex frequency $\Omega_H(n_z)$ reads for $n_x^2(n_z) > 0$

$$\Omega_H(n_z) \approx \pm \delta_0 n_x(n_z) \left(1 + \frac{i}{2} \gamma(n_z)\right)$$

and for $n_x^2(n_z) < 0$

$$\Omega_H(n_z) \approx \pm i\delta_0 |n_x(n_z)| \left(1 + \frac{i}{2} \gamma(n_z)\right)$$

where $\delta_0 = \frac{\omega a}{c}$.

By imposing the boundary conditions at the antenna (vacuum)-plasma interface $\xi = 0$, and to avoid divergence at $\xi = \infty$ (regularity condition) we finally have

$$\tilde{E}_y(\xi, n_z) = \tilde{E}_y^{\text{antenna}}(n_z) e^{i\delta_0 n_x(n_z)\xi} e^{-\frac{\delta_0}{2} n_x(n_z)\gamma(\xi, n_z)\xi} \quad (5.3)$$

when $n_x^2(n_z) > 0$ (propagative case) and

$$\tilde{E}_y(\xi, n_z) = \tilde{E}_y^{\text{antenna}}(n_z) e^{-\delta_0 |n_x(n_z)|\xi} e^{-\frac{i\delta_0}{2} |n_x(n_z)|\gamma(n_z)\xi} \quad (5.4)$$

in the case $n_x^2(n_z) < 0$ (evanescent case).

Now we can reconstruct the field in ξ and $\hat{z} = z/a$ (normalized to the plasma radius) coordinates, for both the propagative and evanescent cases, respectively by performing the inverse Fourier transform of equations (5.3) and (5.4):

$$E_y(\xi, \hat{z}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dn_z \tilde{E}_y(\xi, n_z) e^{in_z \hat{z}} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dn_z \tilde{E}_y^{\text{antenna}} \times (n_z) e^{i(\delta_0 n_x(n_z)\xi + n_z \hat{z})} e^{-(\delta_0/2) n_x(n_z)\gamma(n_z)\xi}$$

$$E_y(\xi, \hat{z}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dn_z \tilde{E}_y(\xi, n_z) e^{in_z \hat{z}} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dn_z \tilde{E}_y^{\text{antenna}} \times (n_z) e^{-\delta_0 |n_x(n_z)|\xi} e^{-i(\delta_0/2) |n_x(n_z)|\gamma(n_z)\xi - n_z \hat{z}}.$$

In figure 12(a) a plot in the plane $\hat{x} - \hat{z}$ of the above solution is depicted showing the propagation of the field towards the center of the tokamak, with maximum intensity starting from the antenna and a progressive decrease of the electric field amplitude.

5.2. Validation of the homogeneous case with consistent inputs in COMSOL simulations

We verify the homogeneous case in COMSOL Multiphysics using identical inputs to ensure the simulation results match theoretical expectations. The model was set up with consistent material properties, boundary conditions, and

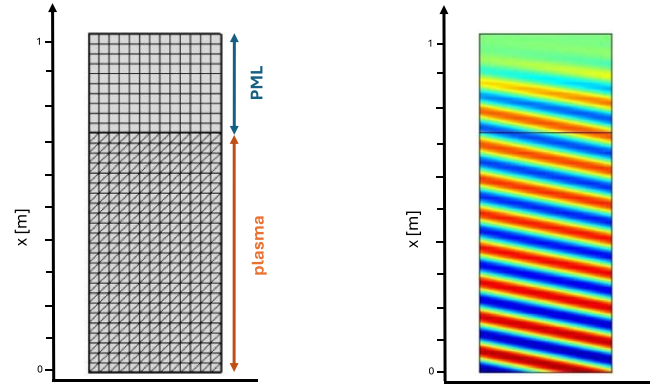


Figure 10. Schematic of the slab structure consisting of a plasma region (plasma width = a) and a Perfectly Matched Layer (PML) region (PML depth = 0.3 m). The x -axis represents the radial coordinate. The discretization of the computational analysis is shown (left). Representation of the $\tilde{E}_y$ component solution obtained from a simulation with $n_z = 9$ and $\epsilon_{yy}^A = 49$ (right).

initial values. Parameters were chosen based on standard references for the homogeneous case ($n_e = 2.11 \times 10^{14} \text{ cm}^{-3}$, $T_e = 10.46 \text{ keV}$, $T_i = 5.92 \text{ keV}$ and $B = 5.4 \text{ T}$) to ensure uniformity.

The geometry was defined to represent a simple plasma slab structure, consisting of a plasma section ($a = 0.727 \text{ m}$) and a Perfectly Matched Layer (PML) section (PML depth = 0.3 m) (see figure 10) [27]. In this simulation, the wave equation equation (2.8) is solved. Each simulation yields a solution in terms of a Fourier transform for a single n_z .

After the simulation, the results are analyzed by constructing a matrix of n_z values and performing an inverse Fourier transform to obtain the field in the spatial domain.

Simulations performed using COMSOL showed excellent agreement with the analytical results, particularly evident when observing the real part of the propagated electric field E_y in the $\hat{x} - \hat{z}$ plane (see figure 11).

The computational time for simulation is approximately 27 min, performed on a computer with an 8-core, 3.20 GHz processor, and 16 GB of RAM. These results confirm the validity of the analytical model set up and demonstrate the reliability of COMSOL.

5.3. Airy case

Another possible analytical solution can be obtained when the complex frequency Ω_{Cx} varies linearly with space. This situation, more realistic than that analyzed above, is compatible with the density increases ($\Omega_{Cx}(\hat{x}) \sim n_e(\hat{x})$ scales essentially with $n_e(\hat{x})$). Putting $\Omega_{Cx}^2(\hat{x}, n_z) = \hat{\Omega}_{Cx}^2(\hat{x} = 0, n_z) (1 - \alpha \hat{x})$ with $\alpha = 1 - \frac{n_x^2(\hat{x}=1, n_z)}{n_x^2(\hat{x}=0, n_z)}$, and introducing the new space coordinate $\xi(\hat{x}, n_z) = \hat{\Omega}_{Cx}^{2/3}(\hat{x} = 0, n_z) (1 - \alpha \hat{x})$, we have that the wave equation reduces to the Airy equation with complex variable

$$\frac{\partial^2 \tilde{E}_y(\xi, n_z)}{\partial \xi^2} + \xi \tilde{E}_y(n_z) = 0 \quad (5.5)$$

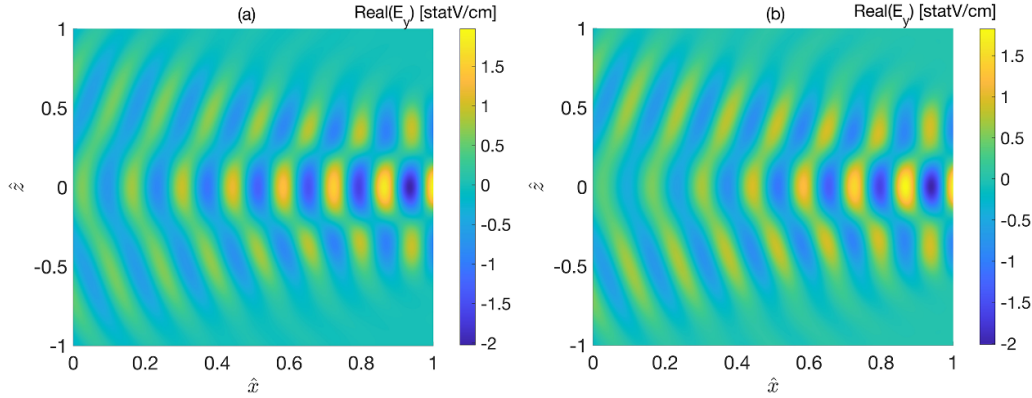


Figure 11. (a) 2D profile of electric field component $\Re(E_y(\hat{x}, \hat{z}))$ (with local plasma parameters $n_e = 2.11 \times 10^{14} \text{ cm}^{-3}$, $T_e = 10.46 \text{ keV}$, $T_i = 5.92 \text{ keV}$ and $B = 5.4 \text{ T}$) propagated by the antenna into the plasma, performed through analytic solution of wave equation (Homogeneous case). Antenna placed at $\hat{x} = 1$ and plasma center at $\hat{x} = 0$. (b) 2D profile of the field component $\Re(E_y(\hat{x}, \hat{z}))$ propagated into by the antenna into the plasma performed by COMSOL.

whose solution can be given in terms of the Airy functions of complex argument ([28], chapter 10).

$$\tilde{E}_y(\xi, n_z) = \tilde{E}_y = C_1 Ai(-\xi) + C_2 Bi(-\xi). \quad (5.6)$$

Also, in this case by imposing the boundary conditions at the antenna (vacuum)-plasma interface $\hat{x} = 1$:

$$\xi_1 = \left[\hat{\Omega}_{Cx}^2(\hat{x} = 0, n_z) - \hat{\Omega}_{Cx}^2(\hat{x} = 1, n_z) \right]^{-2/3} \frac{\hat{\Omega}_{Cx}^2(\hat{x} = 1, n_z)}{\left(\hat{\Omega}_{Cx}^2(\hat{x} = 0, n_z) \right)^{-2/3}} \quad (5.7)$$

and by avoiding the divergence due to the Bi function at $\xi = \infty$ (regularity condition) we have that the entire solution can be written as

$$\tilde{E}_y(\xi, n_z) = \frac{\tilde{E}_y^{\text{antenna}}(n_z) [Ai(-\xi) + iBi(-\xi)]}{Ai(-\xi_1) + iBi(-\xi_1)}. \quad (5.8)$$

Since $\alpha \approx 1$, equation (5.8) becomes

$$\tilde{E}_y(\xi, n_z) = \frac{\tilde{E}_y^{\text{antenna}}(n_z) [Ai(-\xi) + iBi(-\xi)]}{Ai(0) + iBi(0)}. \quad (5.9)$$

Now we can map the electric field in the real normalized space x , and to perform the Fourier transform to obtain the field in space $\xi - \hat{z}$

$$E_y(\xi, \hat{z}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dn_z \tilde{E}_y(\xi, n_z) e^{in_z \hat{z}} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dn_z \tilde{E}_y^{\text{antenna}} \times (n_z) \frac{Ai(-\xi) + iBi(-\xi)}{Ai(0) + iBi(0)} e^{in_z \hat{z}}.$$

At this point, it is possible to revert to in the plane space $\hat{x} - \hat{z}$. In figure 12(c) a surface plot in the plane space $\hat{x} - \hat{z}$ shows the evolution of the antenna electric field into the plasma.

5.4. Weber case

Finally, the last analytical solution can be obtained by considering a parabolic behavior of the complex frequency ω with respect to the $\hat{x}$ variable. To this end looking at the figures 9(a) and (b), the behavior of the real part of $\Omega_{Cx}^2(\hat{x}, n_z)$ and imaginary part of $\Omega_{Cx}^2(\hat{x}, n_z)$ can be fitted in space by a branch of parabola, the real part with a weakly dependence on n_z while the imaginary part with a strong dependence. In the case of first pass absorption the power deposition would be satisfactorily accounted. The differential equation in this case can be re-casted to the Weber equation

$$\frac{d^2 \tilde{E}_y}{d\hat{\chi}^2} + \left[\frac{\hat{\chi}^2}{4} - \hat{k} \right] \tilde{E}_y = 0 \quad (5.10)$$

where the new complex variable

$$\hat{\chi} = \hat{\chi}(\hat{x}, n_z) = \frac{\sqrt{2a}}{1/4} \left[\sqrt{A_c} \hat{x} + \frac{B_c}{2\sqrt{A_c}} \right]$$

depends on both $\hat{x}$ and the complex coefficients A_c and B_c of the parabolic fit (which in turn depend on n_z),

$$\text{and } \hat{k} = \hat{k}(n_z) = \frac{1}{8} a \left(\frac{B_c^2 + 4A_c C_c}{3/2} \right) \text{ is a complex parameter}$$

which depends only on n_z through the fit coefficients.

Solutions of the above equations are combination of confluent hyper-geometric functions

$$\tilde{E}_y(\hat{k}, \hat{\chi}) = C_1 U(\hat{k}, \hat{\chi}) + C_2 V(\hat{k}, \hat{\chi}) \quad (5.11)$$

where $U(\hat{k}, \hat{\chi})$ and $V(\hat{k}, \hat{\chi})$ are parabolic cylinder functions (see [28], chapter 19).

The function $U(\hat{k}, \hat{\chi})$ approaches zero for large values of z and $|\arg(\hat{\chi})| < \pi/2$, while $V(\hat{k}, \hat{\chi})$ diverges for large values of positive real z . The arbitrary constants C_1 and C_2 are fixed by considering the $\hat{\chi}$ large limit (the V function is disregarded because it diverges), and at the plasma edge U is matched with

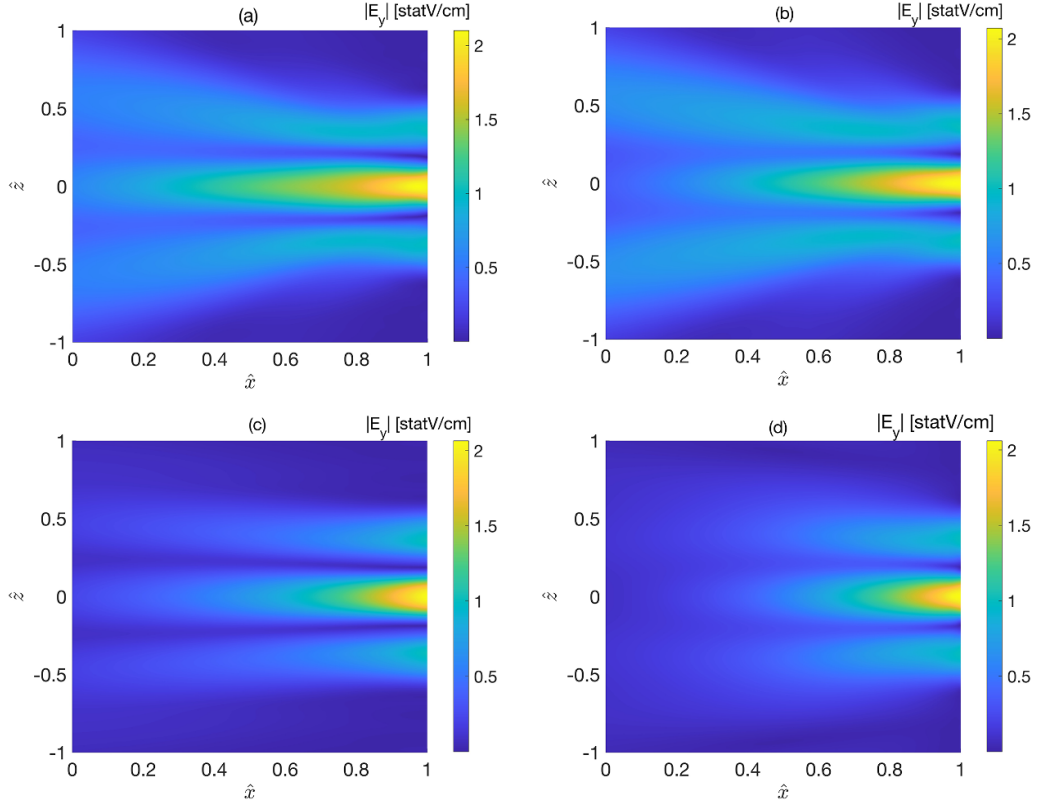


Figure 12. (a) Homogeneous Analytic case: 2D profile of electric field $|E_y(\hat{x}, \hat{z})|$ (with local plasma parameters $n_e = 2.11 \times 10^{14} \text{ cm}^{-3}$, $T_e = 10.46 \text{ keV}$, $T_i = 5.92 \text{ keV}$ and $B = 5.4 \text{ T}$) propagated by the antenna into the plasma, where $\hat{z} = z/a$ with a the minor radius of the plasma. Antenna placed at $\hat{x} = 1$ and plasma center at $\hat{x} = 0$. (b) Homogeneous COMSOL case: 2D profile of electric field $|E_y(\hat{x}, \hat{z})|$ (with local plasma parameters as in (a)) performed by COMSOL. (c) Airy case: 2D profile of electric field $|E_y(\hat{x}, \hat{z})|$ propagated by the antenna into the plasma. (d) Weber case: 2D profile of electric field $|E_y(\hat{x}, \hat{z})|$ propagated by the antenna into the plasma.

the fields at the antenna, i.e.

$$\tilde{E}_y(\hat{k}, \hat{\chi}) = \frac{\tilde{E}_y^{\text{ant}}(\hat{k})}{U(\hat{k}, \hat{\chi}_{\text{ant}})} U(\hat{k}, \hat{\chi}) \quad (5.12)$$

where $\hat{\chi}_{\text{ant}}$ is the value of the new variable evaluated at the antenna location, i.e. $\hat{\chi}_{\text{ant}} = \hat{\chi}(\hat{x} = 1, n_z)$. In figure 12(d) a 2D plot of the field as described in equation (5.11), is shown.

Furthermore, advanced analytical configurations have been investigated to gain deeper insights into the system's behavior. This approach involved dividing the domain into multiple subregions and solving the wave equation within each subdomain. For instance, in regions near the antenna, which are more affected by TTMP damping, the solutions were formulated using the Homogeneous or Airy model. In contrast, in regions closer to the resonance, Weber approach was utilized, as it is particularly effective in capturing the parabolic behavior of $\Omega_{Cx}^2(\hat{x}, n_z)$.

5.5. Quantitative estimation of power absorption in weber analytical model

The analysis carried out to estimate the power absorbed by the plasma in the Weber analytical case is based on the expression, $P_{\text{abs}} = \frac{\omega}{8\pi} \tilde{E}^* \cdot \epsilon^A \cdot \tilde{E}$ ([19], chapter 2). Figure 13 illustrates the

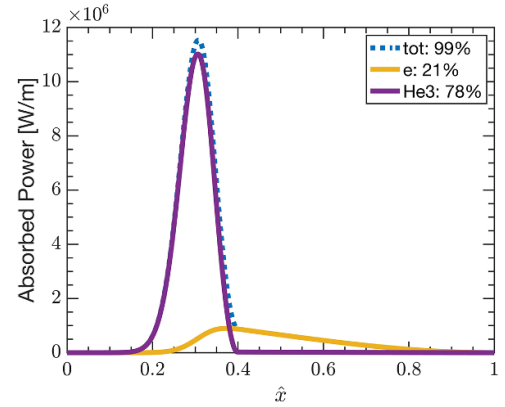


Figure 13. Power deposition profile expressed as power absorbed by the plasma $P_{\text{abs}} [\text{W m}^{-1}]$ as function of $\hat{x}$.

power deposition profile, highlighting the spatial distribution of energy transfer and providing a quantitative estimation of the energy absorbed by the plasma.

For an antenna-launched power of 1 MW, the plasma absorbs approximately 99% of the power, with peak absorption occurring near the resonance of the minority ion species. By dividing the problem domain and solving each subdomain using the Weber approach, it was possible to determine the

power absorbed by the individual plasma species, as shown in figure 13. However, while this analytical method provides a reliable general estimate, a more accurate assessment would require dividing the problem into n domains, favoring a numerical approach and necessitating the use of computational models for higher precision [17].

6. Discussion

In this paper, we explored three semi-analytical solutions for the coupling model of an electromagnetic wave in the IC frequency domain in a tokamak configuration. The solutions presented include the homogeneous case, the Airy case, and the Weber case. The homogeneous case assumes a constant dependence on the kinetic profiles, representing a highly simplified hypothesis that allows the wave solution to be written as the sum of a forward and a reflected wave. Simulations performed using COMSOL closely matched the analytical results, as clearly shown in figure 11 and in figure 12(b) when compared to figure 12(a). COMSOL solves the wave equation, providing a realistic representation of wave propagation in a homogeneous plasma with constant profiles. The Airy and Weber cases aim to provide a more accurate description of the dependence on the kinetic profiles. The Airy case, with its linear dependence on the kinetic profiles, accurately describes the TTMP damping, which is typical of the electronic component. This reflects a realistic behavior of the electromagnetic wave in a plasma with a density gradient, which is compatible with the density increases ($\Omega_C(\hat{x}) \sim n_e(\hat{x})$ scales essentially with $n_e(\hat{x})$). The Weber approach, with its quadratic dependence of the kinetic profiles, provides an even more accurate representation of the $\Omega_C(\hat{x})$, considering not only the presence of TTMP damping but also the ion resonance (see figure 9(b)). The inclusion of the resonance peak in the Weber case results in greater damping of the wave compared to the Airy case, as can be observed in figure 12(d).

Incorporating these cases enables a more accurate approximation of the real complexity of plasma kinetic profiles, offering deeper insights into the behavior of electromagnetic waves under realistic tokamak conditions.

7. Conclusions

A simple one-dimensional model of propagation and absorption of electromagnetic waves in the plasma has allowed us to consider the coupling of the electromagnetic field radiated by the ion cyclotron antenna with the wave that propagates and is absorbed by the plasma. The field radiated by the antenna was studied and calculated with the commercial software CST which considers the detailed geometry of the antenna. The matching of the EM fields is performed at the antenna-vacuum-plasma interface. A simple absorption model is incorporated into the formulation of the wave equation that must be solved in the plasma zone, and it is based on a comparative evaluation of the dielectric tensor. After a Fourier decomposition in parallel (k_z) and vertical (k_y , with $k_y = 0$) wave-numbers a 1D (x slab variable) second order differential

equation for the E_y field radiated by the IC antenna has been obtained in the complex domain of the coefficients by completely neglecting the parallel (to the external magnetic field) component E_z . Analytical solutions can be obtained when the complex coefficient does not depend on space (homogeneous case), when it depends linearly on space (Airy case), and when it has a quadratic dependence on space (Weber case). The reconstruction of the field in z - y direction has been obtained by the inverse Fourier Transform analysis. The calculation is applied to the study of the antenna and plasma scenario of DTT tokamak, particularly the ^3He minority heating scenario at 60 MHz.

This study offers valuable insights into the physics of coupling and propagation through a simplified calculation tool, while also providing a quantitative, though simplified, estimate of the power absorbed by the plasma. This work serves as a foundation for a more detailed investigation that will be conducted using advanced numerical tools. Future developments will extend the current approach by upgrading the governing differential equation from second order to fourth order. This refinement will enable a more accurate treatment of the ion-ion hybrid resonance. Furthermore, the forthcoming studies will incorporate more realistic geometries, such as cylindrical or toroidal configurations, to accurately reflect the actual magnetic field structure and plasma geometry.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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