

Titolo

ADVANCES OF THE HGPT-BU METHODOLOGY FOR BURN-UP ANALYSIS

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 Metodologia HGPT-BU applicata al Burn-up

Sommario

Il presente lavoro ha come obiettivo lo sviluppo di una procedura di calcolo, del tutto generale, in vista di una sua applicazione all'analisi perturbativa e di sensitività del progetto fisico di reattori di nuova generazione, critici o sottocritici. Esso rappresenta la continuazione rispetto a lavori precedenti sullo stesso argomento che hanno in particolare riguardato studi sulle quantità:

- accumulo di un nuclide a fine ciclo di burn-up in una zona di evoluzione
- rapporto di funzionali lineari della densità neutronica a fine ciclo

Nel presente lavoro ci si è concentrati specificatamente sul funzionale 'Parametro di controllo a fine ciclo'. E' stata adottata una modellizzazione del campo neutroni/nuclidi del tutto generale considerando sistemi dai più semplici ai più complessi. Questa nuova modellizzazione potrà essere comunque facilmente estesa a qualsiasi altro funzionale.

Summary

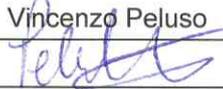
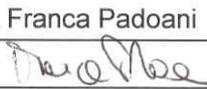
The present work has as its objective the development of a totally general calculation procedure in view of its application to the perturbative and sensitivity analysis of the physical project of new generation reactors, critical or sub-critical. It represents the continuation with respect to previous works on the same subject which have in particular concerned quantity studies:

- accumulation of a nuclide at the end of the burn-up cycle in an evolution zone
- ratio of linear functionals of the neutron density at end of cycle

In the present work, we focused specifically on the functional 'Control parameter at end of cycle'. A general modeling of the neutron / nuclides field has been adopted, considering systems from simple to complex ones. This new modeling may however be easily extended to any other functional.

Note
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1. Introduction

The methodology for the perturbative calculation of functionals of the density of neutrons and of the nuclides evolving during the evolution (burn-up) has been developed on the basis of the Generalized Perturbation Theory founded on the Heuristic Generalized Perturbation Theory (HGPT) [1, 2].

The present work has as its main objective the development of a general calculation procedure in view of its application to the perturbative and sensitivity analysis of physical projects relevant to new generation reactors, at critical or sub-critical conditions. It represents the continuation of previous works on the same topic [3, 4]. In these latter works various validation tests were made making use of the neutronic calculation code Eranos [5].

The cases studied with this methodology concerned in particular the quantities:

- accumulation of a nuclide at the end of the burn-up cycle in an evolution zone
- ratio of functionals linear with the neutron density

In the present work, we focused specifically on the functional 'Control parameter at end of cycle'. A general modeling of the neutron / nuclides field has been adopted considering systems from simplest to complex ones. This modeling may however be easily extended to any other functional.

2. General Theory

In order to derive the equation governing the variables describing the system, a number of assumptions has been made. Specifically:

- The densities of the nuclides that make up the fuel refer to average values relevant to the macro zones in which the core has been subdivided. They are represented with the vector c_z [$z = 1, 2, \dots, Z$ (number of sub-zones)]
- The neutron density (n), dependent on phase space and time, is given in a continuous energy distribution
- The diffusion approximation has been considered (which is widely used for burn-up calculations)

At the densities $n(r,t)$ and $c_z(r)$, defined in the interval (t_o, t_F) , an intensive control variable is associated, $\rho(t)$, such as to maintain constant the total power assigned $W(t)$. The variable $\rho(t)$ may represent, for example, the total degree of insertion of the control rods in the core (not their relative movement, which may be generally described by parameters), or the average density of a neutron poison in the refrigerant. Generally, a fictitious control parameter is normally adopted, i.e., a coefficient that multiplies the fission source. The application of the methodology to different methods of control is however possible, this implying a different normalization to be imposed on a functions associated with the neutron density [1]. In a sub-critical system (ADS), $\rho(t)$ may represent the intensity of the source (via adjustment of the accelerator current intensity). The non-linear equations to which the variables n , c and ρ must satisfy may therefore be written, in the most general case, as:

$$\begin{cases} m_{(n)}(n, \mathbf{c}, \rho | \mathbf{p}) = -\frac{\partial n}{\partial t} + B(\mathbf{c}, \rho | \mathbf{p})n + \delta(t - t_o)n_o = 0 & \text{(reattore critico)} \\ m_{(n)}(n, \mathbf{c}, \rho | \mathbf{p}) = -\frac{\partial n}{\partial t} + B(\mathbf{c}, | \mathbf{p})n + \delta(t - t_o)n_o + \rho s_n(\mathbf{p}) = 0 & \text{(reattore sottocrit.)} \end{cases} \quad (1)$$

$$m_{(c)}(n, \mathbf{c} | \mathbf{p}) = -\frac{\partial \mathbf{c}}{\partial t} + E(n, \mathbf{c} | \mathbf{p})\mathbf{c} + \delta(t - t_o)\mathbf{c}_o + \mathbf{s}_c(\mathbf{p}) = \mathbf{0} \quad (2)$$

$$m_{(\rho)}(n, \mathbf{c} | \mathbf{p}) = \langle \int_{\Delta E} dE \mathbf{c}^T \Gamma \boldsymbol{\sigma}_f n \rangle_{sys} = 0, \quad (3)$$

where $\langle \rangle_{sys}$ indicates integration over the whole volume of the multiplying zone, B represents the operator in diffusion or transport approximation (dependent on \mathbf{c} and, generally, from ρ), the vector \mathbf{p} the system parameters. s_n and s_c are source terms, while

$$\boldsymbol{\sigma}_f = \begin{pmatrix} \sigma_{f,1} \\ \sigma_{f,2} \\ \vdots \\ \sigma_{f,M} \end{pmatrix} \quad \Gamma = \begin{pmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \gamma_M \end{pmatrix} \quad (4)$$

where $\sigma_{f,m}$ is the microscopic fission cross section of the isotope m in group g while γ_m represents the amount of fission energy of the m -th element. The quantities γ_m , W and $\sigma_{f,m}$ are generically represented as system parameters (p_j). The terms in which the Dirac deltas appear are initial conditions.

If, for simplicity, it is desired to replace the constant power condition with the constant fission rate condition, it will be sufficient to set $\gamma_m = 1$.

In the following we will take into consideration the general case of both critical and sub-critical multiplying systems.

The source term s_c in the second member of equation (2) is generally given by a sum of delta functions defined at specific times to account for fuel loads and shuffling operations.

In the equation (1) for the sub-critical case, control over the source was chosen. If the control were in operator B , the methodology would be similar to that for the critical reactor.

In quasi-static problems, such as those involving burn-up studies, the derivative term $\frac{\partial n}{\partial t}$ is negligible. However, its notation is maintained to allow, as we will see later, for the determination of the correct operator that governs the importance function.

A general form of a (linear) response Q may be written in the form

$$Q = \int_{t_0}^{t_F} dt \left(\left\langle \int_{\Delta E} dE \mathbf{h}_n^+ \mathbf{n} \right\rangle_{\text{sys}} + \left\langle \mathbf{h}_c^{+T} \mathbf{c} \right\rangle_{\text{sys}} + \mathbf{h}_p^+ \rho \right) \quad (5)$$

with \mathbf{h}_n^+ , \mathbf{h}_c^+ , \mathbf{h}_p^+ quantities given.

From the linearization procedure, and recalling the coordinate complementation rule [1], we may obtain the linear equations that govern the derivative functions and the importance functions.

The system of equations of the derivative functions is:

$$\left| \begin{array}{ccc} \left(-\frac{\partial}{\partial t} + B \right) & \Omega_n & \chi \int_{\Delta E'} dE' \mathbf{c}^T \mathbf{v} \sigma_f \mathbf{n} \langle \cdot \rangle_{\text{sys}} \\ \Omega_c & \left(-\frac{\partial}{\partial t} + E \right) & 0 \\ \left\langle \int_{\Delta E} dE \mathbf{c}^T \Gamma \sigma_f \langle \cdot \rangle_{\text{sys}} \right\rangle & \left\langle \int_{\Delta E} dE \sigma_f^T \Gamma \langle \cdot \rangle_{\text{sys}} \right\rangle & 0 \end{array} \right| \left| \begin{array}{c} \mathbf{n}_{/j} \\ \mathbf{c}_{/j} \\ \tilde{\rho}_{/j} \end{array} \right| + \frac{\partial}{\partial p_j} \left| \begin{array}{c} \mathbf{m}_{(n)} \\ \mathbf{m}_{(c)} \\ \mathbf{m}_{(\rho)} \end{array} \right| = \mathbf{0} \quad (6)$$

where Ω_n e Ω_c are the coupling operators $\frac{\partial(B\mathbf{n})}{\partial \mathbf{c}}$ e $\frac{\partial(\mathbf{E}\mathbf{c})}{\partial \mathbf{n}}$, respectively¹

By reversing the operators, the importance functions result governed by the equations

$$\left| \begin{array}{ccc} \left(\frac{\partial}{\partial t} + B^* \right) & \Omega_c^* & \int_{\Delta E} dE \sigma_f^T \Gamma \mathbf{c} \langle \cdot \rangle_{\text{sys}} \\ \Omega_n^* & \left(\frac{\partial}{\partial t} + E_i^T \right) & \int_{\Delta E} dE \sigma_f^T \Gamma \langle \cdot \rangle_{\text{sys}} \\ \left\langle \mathbf{v} \sigma_f^T \mathbf{c} \mathbf{n} \int_{\Delta E'} dE' \chi \langle \cdot \rangle_{\text{V,E}} \right\rangle & 0 & 0 \end{array} \right| \left| \begin{array}{c} \mathbf{n}^* \\ \mathbf{c}^* \\ \tilde{\rho}^* \end{array} \right| + \left| \begin{array}{c} \mathbf{h}_n^+ \\ \mathbf{h}_c^+ \\ \mathbf{h}_\rho^+ \end{array} \right| = \mathbf{0} \quad (7)$$

Where Ω_n^* and Ω_c^* are the adjoints of operators Ω_n e Ω_c , respectively.

Setting

$$\rho^* = \langle \tilde{\rho}^* \rangle_{\text{sys}}, \quad (8)$$

the equation corresponding to the first row, concerning the importance of neutrons, is:

$$\left(\frac{\partial}{\partial t} + B^* \right) \mathbf{n}^* + \Omega_c^* \mathbf{c}^* + S^T \Gamma \mathbf{c} \rho^* + \mathbf{h}_n^+ = 0 \quad (9)$$

while the equation corresponding to the second row, concerning the importance of the nuclides, results:

$$-\frac{\partial \mathbf{c}^*}{\partial t} = E^T \mathbf{c}^* + \Omega_n^* \mathbf{n}^* + S \mathbf{n} \rho^* + \mathbf{h}_c^+ \quad (10)$$

The equation corresponding to the third row, relative to the importance associated with the power control, results:

$$\left\langle \mathbf{n} \mathbf{v} \sigma_f^T \mathbf{c} \int_{\Delta E'} dE' \chi \mathbf{n}^* \right\rangle_{\text{V,E,sys}} + \mathbf{h}_\rho^+ = 0 \quad (11)$$

In case it is $h_{\rho}^{+} = 0$ this equation indicates that the neutron importance function in critical systems is orthogonal in the space phase to the neutron density distribution, while in subcritical ones it is orthogonal to the neutron source distribution.

For critical systems it can be said that the neutron importance function is void of the fundamental mode.

Multiplying the equation relative to the first line of (7) by n on the left, and integrating, it is easily obtained:

$$\rho^{*} = \frac{\langle \int_{\Delta E} dE n \mathcal{Q}_c^{*} \mathbf{c}^{*} \rangle_{sys}}{\langle \int_{\Delta E} dE \boldsymbol{\sigma}_f^T \Gamma \mathbf{c}^{*} \rangle_{sys}} \quad (12)$$

The general expression of the variation δQ resulting from a perturbation of the system parameters may be written in the form:

$$\delta Q = \sum_{j=1}^J \delta p_j \int_{t_0}^{t_F} dt \left(\int_{\Delta E} dE n^{*T} \frac{\partial m_n}{\partial p_j} + \mathbf{c}^{*T} \frac{\partial \mathbf{m}_c}{\partial p_j} + \rho^{*} \frac{\partial m_p}{\partial p_j} \right). \quad (13)$$

¹ The terms $\frac{\bar{\partial}(Bn)}{\partial \mathbf{c}}$ and $\frac{\bar{\partial}(\mathbf{E} \mathbf{c})}{\partial n}$ represent Frechet derivatives [6] (applicable to expressions in which there appear linear operators).

3. Control at end of cycle

In the following we shall consider the application of the HGPT-BU methodology in relation to the functional ‘Control parameter at end of cycle’. For its full comprehension, we shall start from very simple cases up to most general ones. There is no difficulty in transferring the methodology to any other functional defined in the non-linear neutron/nuclide field.

3.1. Simple example

Consider a homogeneous bare system (slab, sphere, or infinite cylinder) with one fissile nuclide of density c , volume V and monoenergetic neutron density n . During the nuclide evolution the global fission rate density (W) is maintained constant by an intensive control function (ρ), fictitiously assumed as a coefficient of the neutron fission source.

One energy group cross-sections are considered.

The functional to be calculated is the control function at end of cycle, i.e. $\rho(t_F)$.

The equations relevant to functions n , c and ρ result, setting for the fissile nuclide $\sigma_c = \sigma_a + \sigma_f$ and defining with Σ_c the macroscopic capture cross section associated with all other elements

$$\left\{ \begin{array}{l} -\frac{dn}{dt} + DV^2 n + (\rho v \sigma_f - \sigma_c) cn - \Sigma_c n = 0 \\ -\frac{dc}{dt} - n \sigma_c c + \delta(t - t_o) c_{1,o} = 0 \\ W - \langle \sigma_f cn \rangle = 0 \end{array} \right. \quad (14)$$

Replacing in the first equation the leakage term with $-DB^2 n$ and the densities c and n with their averages \bar{c} and \bar{n} , we may write:

$$\left\{ \begin{array}{l} -\frac{d\bar{n}}{dt} - DB^2 \bar{n} + (\rho v \sigma_f - \sigma_c) \bar{c} \bar{n} - \Sigma_c \bar{n} = 0 \\ -\frac{d\bar{c}}{dt} - \bar{n} \sigma_c \bar{c} + \delta(t - t_o) \bar{c}_{1,o} = 0 \\ W - V \sigma_f \bar{c} \bar{n} = 0 \end{array} \right. \quad (15)$$

with solutions:

$$\bar{n} = \frac{W}{V \sigma_f \bar{c}}, \quad \bar{c} = \bar{c}_o - \frac{\sigma_c}{\sigma_f} \frac{W}{V} (t - t_o), \quad \rho = \frac{\sigma_c \bar{c} + DB^2 + \Sigma_c}{v \sigma_f \bar{c}} \quad (16)$$

Deriving ρ with respect to \bar{c}_o at time t_F gives directly the sensitivity coefficient:

$$\frac{d\rho(t_F)}{d\bar{c}_o} = -\frac{DB^2 + \Sigma_c}{v\sigma_f \bar{c}^2(t_F)} \quad (17)$$

For this elementary case the equations governing the importance functions result²:

$$\begin{bmatrix} \left[\frac{d}{dt} + D\nabla^2 + (\rho v\sigma_f - \sigma_c)\bar{c} - \Sigma_c \right] & -\bar{c}\sigma_c \frac{\langle . \rangle}{V} & \sigma_f \bar{c} \langle . \rangle \\ (\rho v\sigma_f - \sigma_c) \frac{\langle n(.) \rangle}{V} & \left(\frac{d}{dt} - \bar{n}\sigma_c \right) \frac{\langle . \rangle}{V} & \bar{n}\sigma_f \langle . \rangle \\ \langle v\sigma_f \bar{c}n(.) \rangle & 0 & 0 \end{bmatrix} \begin{bmatrix} n^* \\ c^* \\ \tilde{\rho}^* \end{bmatrix} + \delta(t - t_F) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 \quad (18)$$

The equation relevant to the third row gives:

$$\langle v\sigma_f \bar{c}_i n_i^* \rangle + \delta(t - t_F) = 0 \rightarrow n^* = -\delta(t - t_F)\phi^* \quad (19)$$

which shows how at the final time t_F the neutron importance n_i^* is a singular function formed by a normalized standard neutron adjoint ϕ^* multiplied by the Dirac's function. We may then write:

$$n^* = \begin{cases} -\delta(t - t_F)\phi^*(t) & \text{for } t = t_F \\ 0 & \text{for } t < t_F \end{cases} \quad (20)$$

From the equation corresponding to the first row we may easily verify that, for $t < t_F$, it is $(-\sigma_c c_i^* + \sigma_f \rho^*) = 0$. Substituting the expression of n^* in the equation relevant to c^* , allows to obtain the (constant) solution

$$\bar{c}^* = -\frac{\rho v\sigma_f - \sigma_c}{v\sigma_f \bar{c}(t_F)} \equiv -\frac{DB^2 + \Sigma_c}{v\sigma_f \bar{c}^2(t_F)} \quad (21)$$

which corresponds to the analytical solution obtained directly, Eq.(17).

² Here we cannot replace the leakage term with $-DB^2$ since no space behaviour of the neutron importance has been yet established.

The constancy over time of function \bar{c}^* may be easily interpreted. In fact, due to the constraint over the power (in our case, the global fission rate), a fuel nuclide inserted at any point during the time interval considered would be found at the end of cycle, with corresponding consequent equal reduction of ρ .

3.2. Two nuclides

Consider the same case above with two nuclides, the first one transmuting into the second one. The system of equations becomes:

$$\left\{ \begin{array}{l} -\frac{dn}{dt} + D\nabla^2 n + [(\rho v \sigma_{f,1} - \sigma_{c,1})c_1 + (\rho v \sigma_{f,2} - \sigma_{c,2})c_2]n - \Sigma_c n = 0 \\ -\frac{dc_1}{dt} - n\sigma_{c,1}c_1 + \delta(t - t_o)c_{1,o} = 0 \\ -\frac{dc_2}{dt} - n\sigma_{c,2}c_2 + n\sigma_{a,1}c_1 + \delta(t - t_o)c_{2,o} = 0 \\ W - (\sigma_{f,1}c_1 + \sigma_{f,2}c_2)n \geq 0 \end{array} \right. \quad (22)$$

For this elementary case the equations governing the importance functions result³

$$\left[\begin{array}{l} \frac{d}{dt} + D\nabla^2 + [(\rho v \sigma_{f,1} - \sigma_{c,1})\bar{c}_1 + (\rho v \sigma_{f,2} - \sigma_{c,2})\bar{c}_2] - \Sigma_c \\ (\rho v \sigma_{f,1} - \sigma_{c,1}) \frac{\langle n(\cdot) \rangle}{V} \\ (\rho v \sigma_{f,2} - \sigma_{c,2}) \frac{\langle n(\cdot) \rangle}{V} \\ \langle (v \sigma_{f,1} \bar{c}_1 + v \sigma_{f,2} \bar{c}_2) n(\cdot) \rangle \end{array} \right] \begin{array}{l} -\bar{c}_1 \sigma_{c,1} \frac{\langle \cdot \rangle}{V} \\ (\frac{d}{dt} - \bar{n} \sigma_{c,1}) \frac{\langle \cdot \rangle}{V} \\ 0 \\ 0 \end{array} \begin{array}{l} -\bar{c}_2 \sigma_{c,2} \frac{\langle \cdot \rangle}{V} \\ \bar{n} \sigma_{a,1} \bar{c}_1 \\ (\frac{d}{dt} - \bar{n} \sigma_{c,2}) \frac{\langle \cdot \rangle}{V} \\ 0 \end{array} \begin{array}{l} (\sigma_{f,1} \bar{c}_1 + \sigma_{f,2} \bar{c}_2) \langle \cdot \rangle \\ \bar{n} \sigma_{f,1} \langle \cdot \rangle \\ \bar{n} \sigma_{f,2} \langle \cdot \rangle \\ 0 \end{array} \left. \begin{array}{l} n^* \\ c_1^* \\ c_2^* \\ \rho^* \end{array} \right] + \delta(t - t_F) \begin{array}{l} 0 \\ 0 \\ 0 \\ 1 \end{array} = 0 \quad (23)$$

³ Here we cannot replace the leakage term with $-DB^2$ since no space behaviour of the neutron importance has been yet established.

The equation relevant to the fourth row gives:

$$\langle n(v\sigma_{f,1}\bar{c}_1 + v\sigma_{f,2}\bar{c}_2)n^* \rangle + \delta(t - t_F) = 0 \rightarrow n_I^* = -\delta(t - t_F)\phi^* \quad (24)$$

which shows how at the final time t_F the neutron importance n_i^* is a singular function formed by a normalized standard neutron adjoint flux ϕ^* (with negative sign) multiplied by the Dirac's function.

We may then write:

$$n^* = \begin{cases} -\delta(t - t_F)\phi_F^* & \text{for } t = t_F \\ 0 & \text{for } t < t_F \end{cases} \quad (25)$$

The equations relevant to second and third rows then result:

$$\begin{cases} \frac{d\bar{c}_1^*}{dt} - \frac{1}{V} \frac{(\rho v\sigma_{f,1} - \sigma_{c,1})}{v\sigma_{f,1}\bar{c}_{1,F} + v\sigma_{f,2}\bar{c}_{2,F}} \delta(t - t_F) + \bar{n}\sigma_{a,1}\bar{c}_1^* - \bar{n}\sigma_{c,1}\bar{c}_1^* + \bar{n}\sigma_{f,1}\rho^* = 0 \\ \frac{d\bar{c}_2^*}{dt} - \frac{1}{V} \frac{(\rho v\sigma_{f,2} - \sigma_{c,2})}{v\sigma_{f,1}\bar{c}_{1,F} + v\sigma_{f,2}\bar{c}_{2,F}} \delta(t - t_F) - \bar{n}\sigma_{c,2}\bar{c}_2^* + \bar{n}\sigma_{f,2}\rho^* = 0 \end{cases} \quad (26)$$

$$-\frac{1}{V} \frac{\rho v\sigma_{f,1} - \sigma_{c,1}}{v\sigma_{f,1}\bar{c}_1 + v\sigma_{f,2}\bar{c}_2}$$

As well known, the coefficients of the delta functions correspond to the 'final' conditions for functions \bar{c}_1^* and \bar{c}_2^* . So we have, at t_F :

$$\begin{cases} \bar{c}_1^* = -\frac{1}{V} \frac{\rho v\sigma_{f,1} - \sigma_{c,1}}{v\sigma_{f,1}\bar{c}_1 + v\sigma_{f,2}\bar{c}_2} \\ \bar{c}_2^* = -\frac{1}{V} \frac{(\rho v\sigma_{f,2} - \sigma_{c,2})}{v\sigma_{f,1}\bar{c}_{1,F} + v\sigma_{f,2}\bar{c}_{2,F}} \end{cases} \quad (27)$$

From the equation relevant to the first row, for $t < t_F$, we obtain:

$$\rho^* = \frac{\bar{c}_1\sigma_{c,1}\bar{c}_1^* + (\bar{c}_2\sigma_{c,2} - \bar{c}_1\sigma_{a,1})\bar{c}_2^*}{\sigma_{f,1}\bar{c}_1 + \sigma_{f,2}\bar{c}_2} \quad (28)$$

So, for $t < t_F$, we may define the governing equations:

$$\begin{cases} \frac{d\bar{c}_1^*}{dt} - \bar{n}\sigma_{c,1}\bar{c}_1^* + \bar{n}\sigma_{a,1}c_1\bar{c}_2^* + \bar{n}\sigma_{f,1}\rho^* = 0 \\ \frac{d\bar{c}_2^*}{dt} - \bar{n}\sigma_{c,2}\bar{c}_2^* + \bar{n}\sigma_{f,2}\rho^* = 0 \end{cases} \quad (29)$$

3.3. Two nuclides and continuous energy

Consider the same case above with two nuclides and a continuous energy schematization for the neutron density and cross-sections. Rewriting the equations in terms of a group-wise notation will be straightforward. The governing equations will result, with σ_t indicating the total microscopic cross-section:

$$\begin{cases} -\frac{dn}{dt} + DV^2n - \left(\sigma_{t,1} \frac{\langle c_1 \rangle}{V} + \sigma_{t,2} \frac{\langle c_2 \rangle}{V} \right) n + \frac{\langle c_1 \rangle}{V} \int_{\Delta E'} dE' \sigma_{s,1}^{E' \rightarrow E} n(E') + \frac{\langle c_2 \rangle}{V} \int_{\Delta E'} dE' \sigma_{s,2}^{E' \rightarrow E} n(E') \\ + \rho \frac{\langle c_1 \rangle}{V} \chi \int_{\Delta E'} dE' n \nu \sigma_{f,1} + \rho \frac{\langle c_2 \rangle}{V} \chi \int_{\Delta E'} dE' n \nu \sigma_{f,2} + \int_{\Delta E'} dE' \Sigma_s^{E' \rightarrow E} n(E') - \Sigma_t n = 0 \\ -\frac{d}{dt} \frac{\langle c_1 \rangle}{V} - \frac{\langle c_1 \rangle}{V} \int_{\Delta E} dE \frac{\langle \sigma_{c,1} n \rangle}{V} + \delta(t - t_o) c_{1,o} = 0 \\ -\frac{d}{dt} \frac{\langle c_2 \rangle}{V} - \frac{\langle c_2 \rangle}{V} \int_{\Delta E} dE \frac{\langle \sigma_{c,2} n \rangle}{V} + \frac{\langle c_2 \rangle}{V} \int_{\Delta E} dE \frac{\langle \sigma_{a,1} n \rangle}{V} + \delta(t - t_o) c_{3,o} = 0 \\ W - \left\langle \frac{\langle c_1 \rangle}{V} \int_{\Delta E} dE \sigma_{f,1} n + \frac{\langle c_2 \rangle}{V} \int_{\Delta E} dE \sigma_{f,2} n \right\rangle = 0 \end{cases} \quad (30)$$

Recalling the coordinates complementation rule, according to which the control ρ is replaced by $\langle \tilde{\rho} \rangle_{V,E}$, the quantities \bar{c}_1 and \bar{c}_2 by $\langle \tilde{c}_1 \rangle_E$ and $\langle \tilde{c}_2 \rangle_E$, respectively, and setting $\bar{n}\bar{\sigma}_x$ in place

of $\frac{\langle \sigma_x n \rangle_{V,E}}{V}$, with

$$\bar{n} = \frac{\langle n \rangle_{V,E}}{V}, \quad \bar{\sigma}_x = \frac{\langle \sigma_x n \rangle_{V,E}}{\langle n \rangle_{V,E}}, \quad (31)$$

the equations governing the derivative functions will then result:

$$\begin{aligned}
 & \left[\begin{aligned}
 & \frac{d}{dt} + D\nabla^2 + \rho \left[\bar{c}_1 v \sigma_{s,1} \int_{\Delta E'} dE' \chi(\cdot) + \bar{c}_2 v \sigma_{f,2} \int_{\Delta E'} dE' \chi(\cdot) \right] \\
 & + \bar{c}_1 \int_{\Delta E'} dE' \sigma_{s,1}^{E \rightarrow E'}(\cdot) + \bar{c}_2 \int_{\Delta E'} dE' \sigma_{s,2}^{E \rightarrow E'}(\cdot) \\
 & - \left(\sigma_{t,1} \bar{c}_1 + \sigma_{t,2} \bar{c}_2 \right) + \int_{\Delta E'} dE' \Sigma_s^{E \rightarrow E'} n(E') - \Sigma_t
 \end{aligned} \right] \left[\begin{aligned}
 & - \bar{c}_1 \sigma_{c,1} \frac{\langle \cdot \rangle_{V,E}}{V} \quad (\sigma_{a,1} c_1 - \sigma_{c,2} c_2) \frac{\langle \cdot \rangle_{V,E}}{V} \quad (\bar{c}_1 \sigma_{f,1} + \bar{c}_2 \sigma_{f,2}) \langle \cdot \rangle_{V,E} \\
 & \left(\left[\begin{aligned}
 & \left[\begin{aligned}
 & \rho \bar{n} v \bar{\sigma}_{f,1} \int_{\Delta E'} dE' \chi(\cdot) + \int_{\Delta E'} dE' \frac{\langle n \sigma_{s,1}^{E \rightarrow E'}(\cdot) \rangle_{V,E}}{V} \\
 & - \frac{\langle \sigma_{t,1} n(\cdot) \rangle_{V,E}}{V}
 \end{aligned} \right] \\
 & \left[\begin{aligned}
 & \rho \bar{n} v \bar{\sigma}_{f,2} \int_{\Delta E'} dE' \chi(\cdot) + \int_{\Delta E'} dE' \frac{\langle n \sigma_{s,2}^{E \rightarrow E'}(\cdot) \rangle_{V,E}}{V} \\
 & - \frac{\langle \sigma_{t,2} n(\cdot) \rangle_{V,E}}{V}
 \end{aligned} \right]
 \end{aligned} \right) \left(\begin{aligned}
 & \left(\frac{d}{dt} - \bar{n} \bar{\sigma}_{c,1} \right) \frac{\langle \cdot \rangle_{V,E}}{V} \quad \bar{n} \bar{\sigma}_{a,1} \frac{\langle \cdot \rangle_{V,E}}{V} \\
 & 0 \quad \left(\frac{d}{dt} - \bar{n} \bar{\sigma}_{c,2} \right) \frac{\langle \cdot \rangle_{V,E}}{V}
 \end{aligned} \right) \left[\begin{aligned}
 & \bar{n} \bar{\sigma}_{f,1} \langle \cdot \rangle_{V,E} \\
 & \bar{n} \bar{\sigma}_{f,2} \langle \cdot \rangle_{V,E}
 \end{aligned} \right] \\
 & \left(\left[\left\langle \left(v \sigma_{f,1} \bar{c}_1 + v \sigma_{f,2} \bar{c}_2 \right) n \int_{\Delta E'} dE' \chi(\cdot) \right\rangle_{V,E} \right] \right) \quad \left[\begin{aligned}
 & 0 \quad 0 \\
 & 0
 \end{aligned} \right]
 \end{aligned} \right] \left[\begin{aligned}
 & n^* \\
 & \bar{c}_1^* \\
 & \bar{c}_2^* \\
 & p^*
 \end{aligned} \right]
 \end{aligned}$$

$$+ \delta(t - t_F) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

(33)

The equation corresponding to the third row results:

$$\left\langle n \left(v \sigma_{s,1} \bar{c}_1 + v \sigma_{f,2} \bar{c}_2 \right) \int_{\Delta E'} dE' \chi n^* \right\rangle_{V,E} = -\delta(t - t_F) \quad (34)$$

which shows how at the final time t_F the neutron importance vector n^* is a singular function formed by the normalized standard neutron adjoint flux (with negative sign) multiplied by the Dirac's function, i.e.:

$$\mathbf{n}^* = -\delta(t - t_F)\phi^* \quad (35)$$

For $t < t_F$ the equation corresponding to the first row, considering that \mathbf{n}^* is identically equal to zero, reduces to:

$$-\bar{c}_1\sigma_{c,1}\bar{c}_1^* + \bar{c}_2(\sigma_{a,1}c_1 - \sigma_{c,2}c_2)\bar{c}_2^* + (\bar{c}_1\sigma_{f,1} + \bar{c}_2\sigma_{f,2})\rho^* = 0 \quad (36)$$

Multiplying by \mathbf{n} and integrating over energy, we may then write:

$$\rho^* = \frac{\langle \mathbf{n}[\bar{c}_1\sigma_{c,1}\bar{c}_1^* + (\sigma_{c,2}c_2 - \sigma_{a,1}c_1)\bar{c}_2^*] \rangle_{V,E}}{\langle \mathbf{n}(\bar{c}_1\sigma_{f,1} + \bar{c}_2\sigma_{f,2}) \rangle_{V,E}} \quad (37)$$

The equations corresponding to the second row, recalling the solution (35) for \mathbf{n}^* result:

$$\left\{ \begin{aligned} \frac{d\bar{c}_1^*}{dt} - \left[\rho \frac{\langle n\nu\sigma_{f,1}\phi^* \rangle_{V,E}}{V} + \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,1}^{E \rightarrow E'}\phi^* \rangle_{V,E}}{V} - \frac{\langle n\sigma_{t,1}\phi^* \rangle_{V,E}}{V} \right] \delta(t - t_F) - \bar{n}\bar{\sigma}_{c,1}\bar{c}_1^* + \bar{n}\bar{\sigma}_{a,1}\bar{c}_2^* + \bar{n}\bar{\sigma}_{f,1}\rho^* &= 0 \\ \frac{d\bar{c}_2^*}{dt} - \left[\rho \frac{\langle n\nu\sigma_{f,2}\phi^* \rangle_{V,E}}{V\Delta E} + \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,2}^{E \rightarrow E'}\phi^* \rangle_{V,E}}{V} - \frac{\langle n\sigma_{t,2}\phi^* \rangle_{V,E}}{V} \right] \delta(t - t_F) - \bar{n}\bar{\sigma}_{c,2}\bar{c}_2^* + \bar{n}\bar{\sigma}_{f,2}\rho^* &= 0 \end{aligned} \right. \quad (38)$$

with ρ^* given by equation (37). As well known, the coefficients of the delta functions correspond to the ‘final’ conditions for functions \bar{c}_1^* and \bar{c}_2^* . So we have, at t_F :

$$\left\{ \begin{aligned} \bar{c}_1^* &= -\rho \frac{\langle n\nu\sigma_{f,1}\phi^* \rangle_{V,E}}{V} - \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,1}^{E \rightarrow E'}\phi^* \rangle_{V,E}}{V} + \frac{\langle n\sigma_{t,1}\phi^* \rangle_{V,E}}{V} \\ \bar{c}_2^* &= -\rho \frac{\langle n\nu\sigma_{f,2}\phi^* \rangle_{V,E}}{V} - \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,2}^{E \rightarrow E'}\phi^* \rangle_{V,E}}{V} + \frac{\langle n\sigma_{t,2}\phi^* \rangle_{V,E}}{V} \end{aligned} \right. \quad (39)$$

Since ϕ^* is the standard adjoint flux, the values of $\bar{c}_1^*(t_F)$ and $\bar{c}_2^*(t_F)$ may be obtained following a procedure similar to that used for calculating a standard reactivity worth ($\Delta k_{\text{eff}}/k_{\text{eff}}$), where the

‘perturbation’ in this case would correspond to a single atom of nuclide c_1 , or c_2 in the whole volume considered⁴.

For $t < t_F$ the governing equations would result:

$$\begin{cases} \frac{d\bar{c}_1^*}{dt} - \bar{\sigma}_{c,1}\bar{c}_1^* + \bar{\sigma}_{a,1}\bar{c}_2^* + \bar{\sigma}_{f,1}\rho^* = 0 \\ \frac{d\bar{c}_2^*}{dt} - \bar{\sigma}_{c,2}\bar{c}_2^* + \bar{\sigma}_{f,2}\rho^* = 0 \end{cases} \quad (40)$$

3.4. Two nuclides, continuous energy and Z zones

Consider a case with 2 nuclides, continuous energy and the core subdivided into a number (Z) of active zones into which the densities of the fuel nuclides will be assumed space-averaged.

The system of the equations governing the derivative functions in volume z results:

⁴ In practical cases the reactivity worth considered would be that corresponding to the real nuclide density divided by the density itself.

$$\left[\begin{array}{l} -\frac{d}{dt} + D\nabla^2 + \rho \left[\begin{array}{l} \bar{c}_1 \chi \int_{\Delta E'} dE' v \sigma_{s,1}(\cdot) \\ + \bar{c}_2 \chi \int_{\Delta E'} dE' v \sigma_{f,2}(\cdot) \end{array} \right] \\ + \bar{c}_1 \int_{\Delta E'} dE' \sigma_{s,1}^{E' \rightarrow E}(\cdot) + \bar{c}_2 \int_{\Delta E'} dE' \sigma_{s,2}^{E' \rightarrow E}(\cdot) \\ - (\sigma_{t,1} \bar{c}_1 + \sigma_{t,2} \bar{c}_2) + \int_{\Delta E'} dE' \Sigma_s^{E' \rightarrow E} n(E') - \Sigma_t \end{array} \right] \left[\begin{array}{l} \left(\left[\begin{array}{l} \rho \chi \int_{\Delta E'} dE' v \sigma_{f,1} n(E') \\ + \int_{\Delta E'} dE' \sigma_{s,1}^{E' \rightarrow E} n(E') \end{array} \right] \right) \left(\left[\begin{array}{l} \rho \chi \int_{\Delta E'} dE' v \sigma_{f,2} n(E') \\ + \int_{\Delta E'} dE' \sigma_{s,2}^{E' \rightarrow E} n(E') \end{array} \right] \right) \\ - \sigma_{t,1} n \\ \times \frac{\langle \sigma_{a,1} n \rangle_{V_z, E}}{V_z} \quad \times \frac{\langle \cdot \rangle_{V_z, E}}{V_z} \end{array} \right] \left[\begin{array}{l} \left(\begin{array}{l} \bar{c}_1 \chi \int_{\Delta E'} dE' v \sigma_{f,1} n \\ + \bar{c}_2 \chi \int_{\Delta E'} dE' v \sigma_{f,2} n \end{array} \right) \langle \cdot \rangle_{V, E} \\ \\ \\ \\ \\ \\ \\ \\ \\ \sum_{z'=1}^Z \left\langle \int_{\Delta E} dE (\bar{c}_1 \sigma_{f,1} + \bar{c}_2 \sigma_{f,2}) \langle \xi_z(\cdot) \rangle \right\rangle_{V_z} \quad \left| \sum_{z'=1}^Z \langle \bar{n} \sigma_{f,1} \langle \cdot \rangle_E \rangle_{V_z} \quad \sum_{z'=1}^Z \langle \bar{n} \sigma_{f,2} \langle \cdot \rangle_E \rangle_{V_z} \right| \end{array} \right]$$

$$\left[\begin{array}{l} -\frac{\langle \sigma_{c,1}(\cdot) \rangle_{V_z, E}}{V_z} \bar{c}_1 \\ \left(\frac{\langle \sigma_{a,1}(\cdot) \rangle_{V_z, E}}{V_z} c_1 \right) \\ -\frac{\langle \sigma_{c,2}(\cdot) \rangle_{V_z, E}}{V_z} c_2 \end{array} \right] \left[\begin{array}{l} \left(-\frac{d}{dt} - \bar{n} \sigma_{c,1} \right) \frac{\langle \xi_z(\cdot) \rangle_{V_z, E}}{V_z \Delta E} \quad 0 \\ \left(\bar{n} \sigma_{a,1} \right) \frac{\langle \cdot \rangle_{V_z, E}}{V_z} \quad \left(-\frac{d}{dt} - \bar{n} \sigma_{c,2} \right) \frac{\langle \xi_z(\cdot) \rangle_{V_z, E}}{V_z} \end{array} \right] \quad 0$$

$$\times \left[\begin{array}{l} n_{/p} \\ \bar{c}_{/p}^1 \\ \bar{c}_{/p}^2 \\ \tilde{p}_{/p} \end{array} \right] + \frac{\partial}{\partial p_j} \left[\begin{array}{l} m_{(n')} \\ m_{(c_1)} \\ m_{(c_2)} \\ m_{(p)} \end{array} \right] = 0$$

(41)

By reversing the operators, the system of equations governing the importance functions becomes:

$$\left[\begin{array}{l}
 \frac{d}{dt} + D\nabla^2 \\
 + \rho(\bar{c}_1 v \sigma_{s,1} + \bar{c}_2 v \sigma_{f,2}) \int_{\Delta E'} dE' \chi(\cdot) \\
 + \bar{c}_1 \int_{\Delta E'} dE' \sigma_{s,1}^{E \rightarrow E'}(\cdot) + \\
 \bar{c}_2 \int_{\Delta E'} dE' \sigma_{s,2}^{E \rightarrow E'}(\cdot) - \\
 (\sigma_{t,1} \bar{c}_1 + \sigma_{t,2} \bar{c}_2) \\
 + \int_{\Delta E'} dE' \Sigma_s^{E \rightarrow E'} n(E') - \Sigma_t
 \end{array} \right] \left[\begin{array}{l}
 -\bar{c}_1 \sigma_{c,1} \frac{\langle \xi_z(\cdot) \rangle_{V_z, E}}{V_z} (\sigma_{a,1} c_1 - \sigma_{c,2} c_2) \frac{\langle \xi_z(\cdot) \rangle_{V_z, E}}{V_z} \left(\bar{c}_1 \sigma_{f,1} + \bar{c}_2 \sigma_{f,2} \right) \langle \cdot \rangle_{V_z, E}
 \end{array} \right]$$

$$\left(\left[\begin{array}{l}
 \rho \frac{\langle n v \sigma_{f,1} \int_{\Delta E'} dE' \chi(\cdot) \rangle_{V_z, E}}{V_z} + \\
 \int_{\Delta E'} dE' \frac{\langle n \sigma_{s,1}^{E \rightarrow E'}(\cdot) \rangle_{V_z, E}}{V_z} \\
 - \frac{\langle \sigma_{t,1} n(\cdot) \rangle_{V_z, E}}{V_z \Delta E}
 \end{array} \right] \left\| \left(\frac{d}{dt} - \bar{n} \sigma_{c,1} \right) \frac{\langle \xi_z(\cdot) \rangle_{V_z, E}}{V_z} \quad (\bar{n} \sigma_{a,1}) \frac{\langle \cdot \rangle_{V_z, E_z}}{V_z} \right\| \right) \left[\begin{array}{l}
 \bar{n} \sigma_{f,1} \langle \cdot \rangle_{V_z, E} \\
 \bar{n} \sigma_{f,2} \langle \cdot \rangle_{V_z, E}
 \end{array} \right]$$

$$\left(\left[\begin{array}{l}
 \rho \frac{\langle n v \sigma_{f,2} \int_{\Delta E'} dE' \chi(\cdot) \rangle_{V_z, E}}{V_z} + \\
 \int_{\Delta E'} dE' \frac{\langle n \sigma_{s,2}^{E \rightarrow E'}(\cdot) \rangle_{V_z, E}}{V} \\
 - \frac{\langle \sigma_{t,2} n(\cdot) \rangle_{V_z, E}}{V_z}
 \end{array} \right] \left\| \left(\frac{d}{dt} - \bar{n} \sigma_{c,2} \right) \frac{\langle \xi_z(\cdot) \rangle_{V_z, E}}{V_z} \right\| \right) \left[\begin{array}{l}
 \bar{n} \sigma_{f,1} \langle \cdot \rangle_{V_z, E} \\
 \bar{n} \sigma_{f,2} \langle \cdot \rangle_{V_z, E}
 \end{array} \right]$$

$$\left\langle n(v \sigma_{f,1} \bar{c}_1 + v \sigma_{f,2} \bar{c}_2) \int_{\Delta E'} dE' \chi(\cdot) \right\rangle_{V_z, E} \quad |0 \ 0| \quad 0$$

$$+ \delta(t - t_F) \left[\begin{array}{l}
 0 \\
 0 \\
 0 \\
 1
 \end{array} \right] = 0$$

(42)

The equation corresponding to the third row results:

$$\left\langle n \left[(v\sigma_{f,1}\bar{c}_1 + v\sigma_{f,2}\bar{c}_2) \int_{\Delta E'} dE' \chi n^* \right] \right\rangle_{V,E} = -\delta(t - t_F) \quad (43)$$

which again shows how at the final time t_F the neutron importance vector n^* is a singular function formed by the normalized standard neutron adjoint flux (with negative sign) multiplied by the Dirac's function, i.e.,

$$n^* = -\delta(t - t_F)\phi^* \quad (44)$$

For $t < t_F$ the equation corresponding to the first row, considering that n^* is identically equal to zero, reduces to:

$$-\bar{c}_1\sigma_{c,1}\bar{c}_1^* + \bar{c}_2(\sigma_{a,1}c_1 - \sigma_{c,2}c_2)\bar{c}_2^* + (\bar{c}_1\sigma_{f,1} + \bar{c}_2\sigma_{f,2})\rho^* = 0 \quad (45)$$

Multiplying by n and integrating over energy, we may then write:

$$\rho^* = \frac{\left\langle n \left[\bar{c}_1\sigma_{c,1}\bar{c}_1^* + (\sigma_{c,2}c_2 - \sigma_{a,1}c_1)\bar{c}_2^* \right] \right\rangle_{V,E}}{\left\langle (\bar{c}_1\sigma_{f,1} + \bar{c}_2\sigma_{f,2}) \right\rangle_{V,E}} \quad (46)$$

The equations corresponding to the second row, recalling the solution (43) for n^* result within each zone z :

$$\left\{ \begin{array}{l} \frac{d\bar{c}_1^*}{dt} - \left[\rho \frac{\langle nv\sigma_{f,1} \int_{\Delta E'} dE' \chi \phi^* \rangle_{V_z,E}}{V_z} + \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,1}^{E \rightarrow E'} \phi^* \rangle_{V_z,E}}{V_z} - \frac{\langle \sigma_{t,1} n \phi^* \rangle_{V_z,E}}{V_z} \right] \delta(t - t_F) \\ - \bar{n}\bar{\sigma}_{c,1}\bar{c}_1^* + \bar{n}\bar{\sigma}_{a,1}\bar{c}_2^* + \bar{\sigma}_{f,1}\rho^* = 0 \\ \frac{d\bar{c}_2^*}{dt} - \left[\rho \frac{\langle nv\sigma_{f,2} \int_{\Delta E'} dE' \chi \phi^* \rangle_{V_z,E}}{V_z} + \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,2}^{E \rightarrow E'} \phi^* \rangle_{V_z,E}}{V_z} - \frac{\langle \sigma_{t,2} n \phi^* \rangle_{V_z,E}}{V_z} \right] \delta(t - t_F) \\ - \bar{n}\bar{\sigma}_{c,2}\bar{c}_2^* + \bar{\sigma}_{f,2}\rho^* = 0 \end{array} \right. \quad (47)$$

with ρ^* given by equation (46). As well known, the coefficients of the delta functions correspond to the 'final' conditions for functions \bar{c}_1^* and \bar{c}_2^* . So we have, at t_F , and at each zone z :

$$\left\{ \begin{aligned} \bar{c}_1^* &= -\rho \frac{\langle nv\sigma_{f,1} \int_{\Delta E'} dE' \chi \phi^* \rangle_{V_z, E}}{V_z} - \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,1}^{E \rightarrow E'} \phi^* \rangle_{V_z, E}}{V_z} + \frac{\langle \sigma_{t,1} n\phi^* \rangle_{V_z, E}}{V_z} \\ \bar{c}_2^* &= -\rho \frac{\langle nv\sigma_{f,2} \int_{\Delta E'} dE' \chi \phi^* \rangle_{V_z, E}}{V_z} - \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,2}^{E \rightarrow E'} \phi^* \rangle_{V_z, E}}{V_z} + \frac{\langle \sigma_{t,2} n\phi^* \rangle_{V_z, E}}{V_z} \end{aligned} \right. \quad (48)$$

As in previous case, the values of $\bar{c}_1^*(t_F)$ and $\bar{c}_2^*(t_F)$ may be obtained following a procedure similar to that used for calculating a standard reactivity worth.

For $t < t_F$ the governing equations would result:

$$\left\{ \begin{aligned} \frac{d\bar{c}_1^*}{dt} - \bar{n}\sigma_{c,1}\bar{c}_1^* + \bar{n}\sigma_{a,1}\bar{c}_2^* + \bar{n}\sigma_{f,1}\rho^* &= 0 \\ \frac{d\bar{c}_2^*}{dt} - \bar{n}\sigma_{c,2}\bar{c}_2^* + \bar{n}\sigma_{f,2}\rho^* &= 0 \end{aligned} \right. \quad (49)$$

3.5. M nuclides and Z zones

Consider the same case of previous section with M nuclides.

The resulting equations may be easily obtained.

For ρ^* , defining with c_m ($m=1, 2, \dots, M$) a nuclide inserted at the beginning of cycle (denoted as 'primitive') and with c_{m_t} ($t=1, \dots, T_m$) one produced from c_m after t transmutations, we may write:

$$\rho^* = \frac{\left\langle \sum_{m=1}^M \left(\bar{n}\bar{c}_m \sigma_{c,m} \bar{c}_m^* - \sum_{t=m+1}^{T_m} \bar{c}_{m_{(t-1)}} \sigma_{a,m_{(t-1)}} \bar{c}_{m_t}^* \right) \right\rangle_{V,E}}{\left\langle \sum_{m=1}^M \left(\bar{c}_m \sigma_{f,m} + \sum_{t=m+1}^{T_m} \bar{c}_{m_t} \sigma_{f,m_t} \right) \right\rangle_{V,E}} \quad (50)$$

while, in each zone z , for the importance relative to each nuclide c_x , at t_F ,

$$\frac{d\bar{c}_x^*}{dt} - \left[\rho \frac{\langle nv\sigma_{f,x} \int_{\Delta E'} dE' \chi \phi^* \rangle_{V_z, E}}{V_z} + \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,x}^{E \rightarrow E'} \phi^* \rangle_{V_z, E}}{V_z} - \frac{\langle \sigma_{t,x} n\phi^* \rangle_{V_z, E}}{V_z} \right] \delta(t - t_F) \quad (51)$$

+ other terms = 0

As well known, the coefficients of the delta functions correspond to the ‘final’ conditions for functions \bar{c}_1^* and \bar{c}_2^* . So we have, at t_F , and at each zone z :

$$\bar{c}_x^* = -\rho \frac{\langle nv\sigma_{f,x} \int_{\Delta E'} dE' \chi \phi^* \rangle_{V_z, E}}{V_z} - \int_{\Delta E'} dE' \frac{\langle n\sigma_{s,x}^{E \rightarrow E'} \phi^* \rangle_{V_z, E}}{V_z} + \frac{\langle \sigma_{t,x} n\phi^* \rangle_{V_z, E}}{V_z} \quad (52)$$

As in previous cases, the value of $\bar{c}_x^*(t_F)$ may be obtained following a procedure similar to that used for calculating a standard reactivity worth.

At $t < t_F$, in relation to each primitive nuclide c_m , we would have:

$$\left\{ \begin{array}{l} \frac{d\bar{c}_m^*}{dt} - \bar{n}\bar{\sigma}_{c,m} \bar{c}_m^* + \bar{n}\bar{\sigma}_{a,m} \bar{c}_{m_1}^* + \bar{n}\bar{\sigma}_{f,m} \rho^* = 0 \\ \frac{d\bar{c}_{m_1}^*}{dt} - \bar{n}\bar{\sigma}_{c,m_1} \bar{c}_{m_1}^* + \bar{n}\bar{\sigma}_{a,m_1} \bar{c}_{m_2}^* + \bar{n}\bar{\sigma}_{f,m_1} \rho^* = 0 \\ \dots\dots\dots \\ \dots\dots\dots \\ \frac{d\bar{c}_{m_{T_m}}^*}{dt} - \bar{n}\bar{\sigma}_{c,m_{T_m}} \bar{c}_{m_{T_m}}^* + \bar{n}\bar{\sigma}_{f,m_{T_m}} \rho^* = 0 \end{array} \right. \quad (53)$$

3.6. Numerical example

We shall consider now a simple example to illustrate the methodology, as that described above at paragraph 3.1. In this case the sensitivity coefficient results:

$$\frac{d\rho(t_F)}{d\bar{c}_o} = - \frac{DB^2 + \Sigma_c}{v\sigma_f \bar{c}^2(t_F)} \quad (54)$$

Assuming a core representative of a lead cooled fast reactor fuelled with Pu-239 and the values given in Table 1, the sensitivity coefficient $\frac{d\rho(t_F)}{d\bar{c}_o}$ obtained analytically, Eq. (54), and by numerical

calculation coincide and result equal to $-2.29E-20$. In case the Pu239 density in the core is increased by 1% at beginning of cycle, this would correspond to a change of the control parameter ρ by -0.85% .

Table 1

Pu239 density	D	B ²	Fission rate	Fuel cycle length
1.08E+20 atoms/cm ³	2.08	8.16E-04	1.42E+11/cm ³ sec	180 days

Appendix

Control modality

In the procedure illustrated in the present work, a power control mechanism (or global fission rate) corresponding to a fictitious parameter (ρ) coefficient of the fission source has been assumed for simplicity. In the mathematical formalism it was represented in the equation that governs the neutron density with the term:

$$\rho \chi \int_{\Delta E'} dE' \mathbf{c}^T \mathbf{v} \boldsymbol{\sigma}_f \mathbf{n}. \quad (\text{A.1})$$

As we have seen above, this criterion corresponds to the orthonormality relationship

$$\left\langle \int_{\Delta E} dE \mathbf{n}^* \frac{\partial \mathbf{B}}{\partial \rho} \mathbf{n} \right\rangle_{\mathbf{v}, \mathbf{E}} \equiv \left\langle \mathbf{n} \mathbf{v} \boldsymbol{\sigma}_{s,l}^T \bar{\mathbf{c}} \int_{\Delta E'} dE' \chi \mathbf{n}^* \right\rangle_{\mathbf{v}, \mathbf{E}} = -\delta(t - t_F) \quad (\text{A.2})$$

The control modality over the fission source may be considered adequate in the majority of cases of interest for studies on the evolution of the core in critical systems during the burn-up. In particular cases, however, different control methods may be of interest, for example, the control on the density of the soluble boron in a water-cooled reactor, or the regulation of a control rod. Consider the latter case. The control can be parametrized by a coordinate, say z_c , which establishes the lower insertion point in the core (assuming that the rod is adjusted from above). The displacement δz_c of the rod may be represented with the corresponding replacement of material in section δz_c : boron in case of insertion, refrigerant otherwise. In mathematical formalism, this mechanism can be represented in the equation that governs the neutron density with a term of the type:

$$-\rho \xi_c \left[H(z - z_c) \int_{\Delta E} dE \Sigma_B \mathbf{n} + \bar{H}(z - z_c) \int_{\Delta E} dE \Sigma_{Pb} \mathbf{n} \right] \quad (\text{A.3})$$

where ξ_c is a parameter equal to 1 within the control rod and zero otherwise, Σ_B and Σ_{Pb} are the macroscopic capture cross sections of boron and lead, respectively, while $H(z - z_c)$ is the Heaviside function (equal to 1 for $z \geq z_c$ and zero otherwise) while \bar{H} is its complement (equal to 1 for $z < z_c$ and zero otherwise). The effect of a control rod displacement is almost exclusively due to the greater or lesser presence of boron in the core, while, due to the quantities involved, the effect due to the displacement of lead may be considered negligible. The term (A.1) can therefore be replaced by the simple term:

$$-\rho \xi_c H(z - z_c) \int_{\Delta E} dE \Sigma_B \mathbf{n} \quad (\text{A.4})$$

To note that, assuming the z -coordinate oriented upwards, a greater insertion downwards would result in a decrease in the value of z_c .

By assuming this power control mechanism, the normalization condition for the adjoint flux at t_F in this case becomes:

$$\langle \xi_c \delta(z - z_c) \int_{\Delta E} dE \phi_F^* \Sigma_B n \rangle_V = 1 \quad (\text{A.5})$$

Consider now the standard adjoint flux ϕ_λ^* and the normalization condition:

$$\langle \int_{\Delta E} dE \phi_\lambda^* F n \rangle_V = 1 \quad (\text{A.6})$$

It easy to verify that the relationship between ϕ_λ^* and ϕ_F^* results:

$$\phi_F^* = (1 + \alpha) \phi_\lambda^* \quad (\text{A.7})$$

with

$$\alpha = \frac{1 - \langle \xi_c \delta(z - z_c) \int_{\Delta E} dE \phi_\lambda^* \Sigma_B n \rangle_V}{\langle \xi_c \delta(z - z_c) \int_{\Delta E} dE \phi_\lambda^* \Sigma_B n \rangle_V} \equiv \frac{1 - S_c \int_{\Delta E} dE \phi_\lambda^*(z_c) \Sigma_B n(z_c)}{S_c \int_{\Delta E} dE \phi_\lambda^*(z_c) \Sigma_B n(z_c)} \quad (\text{A.8})$$

where S_c represents the control rod transverse area.

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