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Influence of rotation on axisymmetric plasma equilibria: double-null DTT scenario

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Abstract

We study the influence of toroidal plasma rotation on some relevant tokamak equilibrium quantities. The Grad–Shafranov equation generalised to the rotating case is analytically solved employing two different representations for the homogenous solution. Using an expression in terms of polynomials, we describe the separatrix shape by a few geometrical parameters, reproducing different plasma scenarios such as double-null and inverse triangularity. In this setting, the introduction of toroidal rotation corresponds to variations on relevant plasma quantities, most notably an enhancement of the poloidal beta. Using a more general expression in terms of Bessel functions, we reconstruct the full plasma boundary of the double-null configuration proposed for the upcoming Divertor Tokamak Test experiment, demonstrating how said configuration is compatible with different values of the plasma velocity.

Keywords: magnetohydrodynamics, magnetic confinement and equilibrium, tokamaks

(Some figures may appear in colour only in the online journal)

1. Introduction

The basic concept at the ground of any operational regime of a tokamak device [1] is the existence of an axisymmetric plasma equilibrium [2]. In a real machine, this equilibrium can exist for a time which is inherently limited by the duration of the discharge. The duration is usually much longer than the characteristic timescale on which magnetohydrodynamic instabilities develop, leading to abrupt losses of confinement, and much shorter than dissipation timescales due to resistivity or other non ideal effects, leading to slow losses of confinement.

The description of a tokamak equilibrium is based on the balance of the ideal MHD forces, i.e. pressure gradients versus magnetic pressure and tension, resulting in the well-known Grad–Shafranov equation (GSE) [3], in which the presence of steady matter flux is neglected. This assumption

can be motivated by the conditions of operation of tokamak machines, which discharge is, in general, associated to a flux-free quasi-ideal plasma. Nonetheless, the emergence of a ‘spontaneous rotation’ in Tokamak devices has been observed since the early nineties [4, 5], both in the toroidal and poloidal directions. Many proposals have been argued in order to explain this phenomenon, which can be interpreted as a result of a self-organization of the plasma in the transition from turbulent to laminar flow. Indeed, the transition between turbulent and laminar regimes is an interchange phenomenon, due to the unavoidable linear and nonlinear instability of the rotating plasma [6, 7].

Another important operation condition of a tokamak leading to important rotation profiles is the heating of the plasma via neutral beam injection: the beam injected in the tangential direction, transferring angular momentum into the plasma, can drive rotation flows inside the configuration [8].

According to these considerations, the inclusion of rotation in the computation of a tokamak equilibrium is a relevant topic

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that may require increasing attention in the years to come. The theory of rotating tokamak equilibria has been developed by many authors (e.g. see [9, 10] and citing articles), with most studies mainly relying on the introduction of a Bernoulli-like function (as in traditional fluid dynamics) in order to generalize the GSE to a plasma with flow, while keeping its mathematical form mostly intact [11].

Here, we investigate the case of a tokamak equilibrium in the presence of a toroidal velocity field and we address its description through the introduction of a generalized pressure function, as in [9]. We first construct simple semi-analytical solutions of the obtained equation, generalizing the well-known Solov'ev scenario [12]. Then, we implement our model to study how the double-null configuration at 5 MA of the Divertor Tokamak Test (DTT) Italian proposal [13] is modified by the presence of toroidal rotation. We are able to characterize the influence of toroidal velocity on some basic plasma quantities, such as the poloidal beta β_{pol} , the plasma current I_p , the profile of the safety factor q , the position of the magnetic axis and the morphology of the separatrix with respect to the isobar surface at zero pressure, taken as the plasma boundary.

Clearly, the introduction of a toroidal rotational field in a Tokamak equilibrium can also be studied via numerical modelling, see for instance [14]. However, our semi-analytical study, based on the construction of a generic solution for a linear GSE, is a powerful tool to establish precise relations among the model parameters. The choice of a linear equilibrium allows to individualize the basic eigenfunctions of the configurational problem and it is justified by the expansion of the unknown functions of the magnetic flux function up to the lowest order of approximation. In this respect, our correlation between the parameter governing the rotation intensity and the β value of the plasma must be regarded as a general feature of the considered family of plasma configurations.

The manuscript is structured as follows. In section 2, we recall the fundamental equations from the known literature, we outline the basis for our study introducing the necessary assumptions, and we provide a convenient form for the particular solution of the equilibrium. In section 3, we solve the homogeneous problem using a purely polynomial expansion of the magnetic flux function ψ . We show how this simple solution is able to represent different plasma scenarios, characterized only by few constraints, and what is the impact of plasma rotation on the equilibrium properties. In section 4, we provide a different, more general solution to the homogeneous problem, which allows for a more precise determination of the plasma separatrix while still maintaining a flexible fitting procedure. We study the specific case of the DTT double-null scenario, illustrating the fitting procedure and the impact of rotation on some relevant equilibrium properties. Concluding remarks follow.

2. Basic equations

The equilibrium of magnetically confined plasmas can be described by few basic equations:

$$\rho(\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P + \mathbf{J} \times \mathbf{B}, \quad (1)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2)$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \quad (3)$$

which express the conservation of the momentum of a fluid with mass density ρ , velocity field \mathbf{v} and pressure P in the presence of self-consistent current density \mathbf{J} and magnetic field \mathbf{B} . Working in cylindrical coordinates (R, ϕ, Z) and assuming axisymmetry, i.e. $\partial_\phi f = 0$ for any quantity f , the magnetic field can be expressed as $\mathbf{B} = \mu_0 I \nabla \phi + \nabla \psi \times \nabla \phi$, in terms of the two scalar functions ψ and I , which are related to the poloidal magnetic flux and toroidal magnetic field, respectively. Furthermore in the static case, $\mathbf{v} = 0$, the equilibrium problem reduces to the well-known Grad-Shafranov equation:

$$\Delta^* \psi = -\mu_0^2 I I' - \mu_0 P' R^2, \quad (4)$$

where $\Delta^* \equiv \partial_R^2 - \partial_R/R + \partial_Z^2$, and the prime denotes differentiation with respect to ψ . The solutions of this equation for a confined plasma correspond to nested tori of constant ψ , and have been extensively studied in the literature [15]. Earliest analytical studies focus on the Solov'ev scenario, in which the right-hand side of the equation is made independent on ψ by the assumptions:

$$\mu_0 P' = S_1, \quad \mu_0^2 I I' = S_2, \quad (5)$$

$$\implies \Delta^* \psi = -S_2 - S_1 R^2, \quad (6)$$

with $S_{1,2} = \text{const}$. Other choices can be made while still preserving the linearity of the equation, like quadratic source function scenario with $P', I I' \sim \psi$, or the dissimilar source function scenario, with $P' \sim \text{const}$, $I I' \sim \psi$.

The analysis is more subtle in the case of a plasma configuration rotating in the toroidal direction with velocity $\mathbf{v} = \omega R^2 \nabla \phi$. In this case equation (4) can be generalised as

$$(\Delta^* \psi + \mu_0^2 I I') \nabla \psi = -\mu_0 R^2 (\nabla P - \rho R \omega^2 \nabla R), \quad (7)$$

where two difficulties arise: the plasma mass density enters the equilibrium balance, and the pressure is no longer a pure function of ψ . However, it is clear from ideal Ohm's law, $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$, combined with stationary Faraday's law, $\nabla \times \mathbf{E} = 0$, that the rotation frequency ω is a new surface function, a result also known as corotation theorem [16]. The set of equations must be closed introducing an equation of state for the fluid, with many possible choices [17]; here we consider the ideal gas law $P = \rho \frac{k_B}{m_i} T$, where k_B is the Boltzmann constant, m_i is the ion mass, and T is the plasma temperature, which can be safely assumed to be a surface function in tokamak equilibrium configurations, due to the high parallel transport in these devices. In view of these assumptions, equation (7) can be rewritten as an equality involving two gradients, $\nabla \psi$ and

$$\nabla \left(\frac{k_B}{m_i} T \log \frac{\rho}{\rho_0} - \frac{\omega^2 R^2}{2} \right) \equiv \nabla \theta. \quad (8)$$

Hence, the newly defined θ must be a function of ψ , and the resulting equation is

$$\Delta^* \psi = -\mu_0^2 II' - \mu_0 R^2 \rho \left[\theta' + R^2 \omega \omega' + \left(1 - \log \frac{\rho}{\rho_0} \right) \frac{k_B}{m_i} T' \right]. \quad (9)$$

This expression is further simplified defining a generalized pressure $P_T(\psi) = \rho_0 \frac{k_B}{m_i} T \exp(\frac{m_i \theta}{k_B T}) = P \exp(-\frac{m_i \omega^2 R^2}{2k_B T})$ which is a source function itself, and coincides with the thermodynamic pressure in the $\omega \rightarrow 0$ limit [9]. Finally we have:

$$\Delta^* \psi = -\mu_0^2 II' - \mu_0 R^2 \left[P_T' + P_T R^2 \left(\frac{m_i \omega^2}{2k_B T} \right)' \right] e^{\frac{m_i \omega^2 R^2}{2k_B T}}, \quad (10)$$

which gives the equilibrium of a rotating plasma once the arbitrary functions I , P_T , ω and T are assigned.

Before continuing our analysis, let us introduce the following normalizations: defining the plasma major radius as R_0 and the toroidal magnetic field at the major radius as B_0 , we normalize length with R_0 , magnetic field with B_0 , magnetic flux with $B_0 R_0^2$, pressure with $B_0^2/2\mu_0$, I with $B_0 R_0/\mu_0$ and current density with $B_0/\mu_0 R_0$. All quantities are to be meant adimensional from now on, unless stated otherwise.

2.1. Solov'ev-like configuration

Similarly to the Solov'ev assumption in the static scenario, we can make the right-hand side of equation (10) independent on ψ by setting

$$\frac{P_T'}{2} = P_1, \quad II' = I_1, \quad \frac{m_i \omega^2 R_0^2}{2k_B T} = M^2, \quad (11)$$

where P_1, I_1, M are assumed as constants. The latter expresses the ratio of plasma velocity to thermal velocity at the plasma major radius, and it serves as a parameter to introduce rotation in the equilibrium computation. The resulting expressions for the equilibrium equation and the relevant quantities are:

$$\Delta^* \psi = -R^2 P_1 e^{M^2 R^2} - I_1, \quad (12)$$

$$P_T(\psi) = 2P_1 \psi, \quad P(\psi, R) = 2P_1 \psi e^{M^2 R^2}, \quad (13)$$

$$I(\psi) = \sqrt{2I_1 \psi + I_0}, \quad \omega(\psi) = \frac{M}{R_0} \sqrt{2k_B T(\psi)/m_i}, \quad (14)$$

where I_0 is an integration constant introduced to take into account the vacuum toroidal magnetic field. We observe that, even though in the presence of plasma motion the pressure is not constant on magnetic surfaces, like in the static case, according to equation (13) we still have $P=0$ on the separatrix just defined. Hence the last magnetic surface and the curve of vanishing plasma matter density coincide, a result which

in general does not hold in the presence of plasma motion (see [18] for a recent analysis, or [19] for a more general approach).

As usual in the theory of linear differential equations, the full solution is given by the sum of a particular solution ψ_P , plus the general homogeneous solution defined by $\Delta^* \psi_H = 0$. It is easy to verify by substitution that the former can be written as:

$$\psi_P = \frac{P_1}{4M^4} \left[1 + M^2 R^2 - e^{M^2 R^2} \right] - \frac{I_1}{2} Z^2. \quad (15)$$

In this form, we naturally recover the static Solov'ev solution $-P_1 R^4/8 - I_1 Z^2/2$ in the $M \rightarrow 0$ limit.

3. Polynomial solution

Concerning the solution of $\Delta^* \psi_H = 0$, the usual strategy is to employ separation of variables and assume an expression like $\psi_H \sim f(R)g(Z)$. The linearity of the equation allows to consider a sum of any number of such terms. For instance, many authors consider polynomials in the Z variable, and in the special case of up-down symmetry, corresponding to even power only, the following representation can be used [20]:

$$\psi_H = \sum_{n=0,2,\dots} \sum_{k=0}^{n/2} f_{n,k}(R) Z^{n-2k}. \quad (16)$$

It can be verified by substitution that the functions $f_{n,k}(R)$ are given recursively by the relations

$$\begin{aligned} (\partial_R^2 - \partial_R/R) f_{n,0} &= 0, \\ (\partial_R^2 - \partial_R/R) f_{n,k} &= -(n-2k+1)(n-2k+2) f_{n,k-1}. \end{aligned} \quad (17)$$

This representation of the homogeneous solution in terms of the lowest even powers of Z is suitable for describing up-down symmetric configurations in terms of the minimum number of parameters. For our first analysis, we consider equation (16) truncated at a maximum index $n=6$, resulting in the following expression:

$$\begin{aligned} \psi_H &= C_0 + C_2 R^2 + C_4 (R^4 - 4R^2 Z^2) \\ &\quad + C_6 (R^6 - 12R^4 Z^2 + 8R^2 Z^4) \\ &\quad + C_8 (R^8 - 24R^6 Z^2 + 48R^4 Z^4 - 64R^2 Z^6/5) \\ &\quad + C_{10} (R^2 + 2Z^2 - 2R^2 \ln(R)) \\ &\quad + C_{12} [-15R^4 + 24R^2 Z^2 + 8Z^4 + 12(R^4 - 4R^2 Z^2) \ln(R)] \\ &\quad + C_{14} [5R^6 - 45R^4 Z^2 + 12R^2 Z^4 + 8Z^6/5 \\ &\quad \quad - 3(R^6 - 12R^4 Z^2 + 8R^2 Z^4) \ln(R)], \end{aligned} \quad (18)$$

where C_i ($i=0,2,\dots,14$) are arbitrary integration constants. Without losing real physical content, and to simplify the fitting procedure, we set C_i to zero for $i \geq 10$, so eliminating all terms $\propto \ln(R)$. The remaining free constants are determined according to the plasma minor radius a , the triangularity δ , the elongation κ and the boundary curvature at the outermost

point c (inversely proportional to the squareness of the plasma boundary), via the conditions:

$$\psi(1-a, 0) = 0, \quad (19)$$

$$\psi(1+a, 0) = 0, \quad (20)$$

$$\psi(1-a\delta, a\kappa) = 0, \quad (21)$$

$$\frac{\partial\psi}{\partial R}(1-a\delta, a\kappa) = 0, \quad (22)$$

$$\frac{\partial^2\psi}{\partial Z^2} \bigg/ \frac{\partial\psi}{\partial R}(1+a, 0) = c. \quad (23)$$

(See figure 1 for a visual reference). Concerning the constants P_1, I_1, I_0 contained in equations (14) and (15), we set their values according to the desired plasma poloidal beta β_p , current I_p and toroidal magnetic field on axis, given by:

$$\beta_{\text{pol}} = \frac{\int P ds}{\int ds} = \frac{P_1}{S} \int R \frac{\psi(R, Z) e^{M^2 R^2}}{B^2} ds, \quad (24)$$

$$I_p = \int J_\phi ds = \int R P_1 e^{M^2 R^2} + \frac{I_1}{R} ds, \quad (25)$$

$$B_\phi(R_0, 0) = \frac{\sqrt{2I_1\psi(R_0, 0) + I_0}}{R_0}. \quad (26)$$

The integrals in the above equations are performed over the confined plasma region inside the magnetic separatrix, defined by $\psi(R, Z) = 0$ and corresponding to the boundary between closed and open magnetic lines. The same region can also be described by the points $(R, Z) \in \{1-a, 1+a\} \times \{-Z_m(R), Z_m(R)\}$, with $\psi(R, Z_m(R)) = 0$. The function $Z_m(R)$ can be calculated explicitly, describing the plasma upper boundary (or lower, with minus sign) in terms of $a, \delta, \kappa, c, P_1, I_1$ and M , however we omit its cumbersome expression for brevity. In general, closed-form expressions for the integrals cannot be found, hence we resort to standard numerical recipes for their calculation. In the practical implementation, we find that a simple *guess and check* strategy leads to satisfying results after a single iteration.

3.1. Study of rotation influence

We note that the rotation velocity is treated as a free parameter so far, via the constant M , with $M = 0$ corresponding to the static plasma case. Within this framework, we are able to evaluate its direct impact on the other equilibrium features. In figure 2 we show the capabilities of solution equation (18) of reproducing plasma shapes with different values of the parameters, as in table 1.

With regard to the second row, the pointy shape of the profile at its top and bottom suggests the presence of x -points.

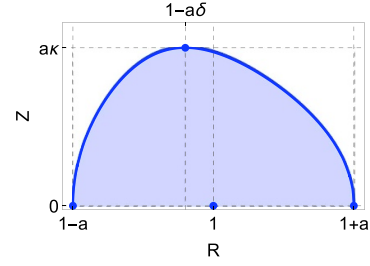


Figure 1. Analytic plasma shape defined as the curve $\psi(R, Z) = 0$, parametrized through the constants a, δ, κ and c , for some arbitrary values of the physical constants P_1, I_1 and M .

Table 1. Values of the minor radius a , triangularity δ , elongation κ and curvature c for the plasma configurations of figure 2.

	a	δ	κ	c
First row	0.25	0.35	1.20	5
Second row	0.30	0.45	1.92	2
Third row	0.25	-0.35	1.80	1

However, it must be noted that the solution used here has not enough free constants to impose the proper null condition on the magnetic field at a desired location; the x -points can only emerge at a certain location, fixed by parameters δ and κ , for specific choices of a and c . We will see in the next section how to address this shortcoming.

It is worth noting that the magnetic flux surfaces are not affected by plasma rotation as much as the isobaric surfaces. This can be explained considering equation (13) describing the plasma pressure, which contains the exponential term $e^{R^2 M^2}$ and thus a more sensitive dependence on the parameter M . Concerning other relevant plasma quantities, figure 3 shows the dependence on M of β_{pol}, I_p and q_{95} , the safety factor at 95% plasma volume, normalized to their respective values in the static case. The other parameters used for the fit correspond to the second row scenario of table 1. While the entity of the variations differ for other choices of the parameters, however, the general qualitative behaviour is consistently that of an enhancement of both β_{pol} and I_p , while the safety factor is reduced. In all the considered cases this has never resulted in breaking the Kruskal-Shafranov stability condition $q > 1$ over the whole plasma profile, even for unrealistically high rotation velocities.

These results suggest that, when modeling real plasma equilibria using a static analytical solution or numerical code, the errors committed can get increasingly large with plasma rotation. The evaluation of M can thus give quantitative insight on the necessity to employ an exact solution or equilibrium solver which take the plasma rotation into account.

4. General solution

Suppose that the plasma boundary curve is known, either analytically or numerically, and one wants an accurate fit reproducing its shape. The polynomial solution equation (18) studied in the previous section is easy to implement, requiring only

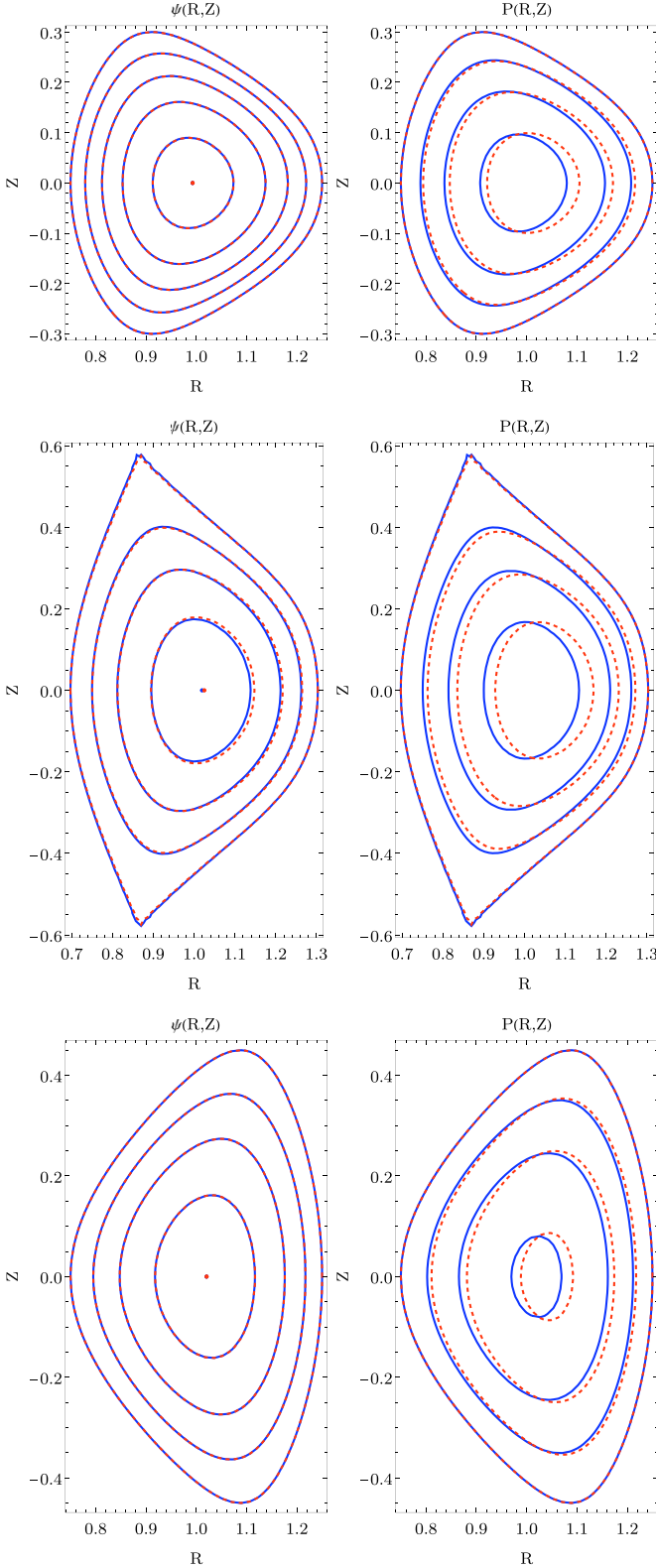


Figure 2. Contours of constant magnetic flux ψ (left) and pressure P (right) in the (R, Z) plane, calculated for the $M=0$ (solid) and $M=0.6$ (dashed) cases and corresponding to the parameters reported in table 1.

few constraints in order to obtain the equilibrium, but it evidently fails in such situations. As discussed above, the position

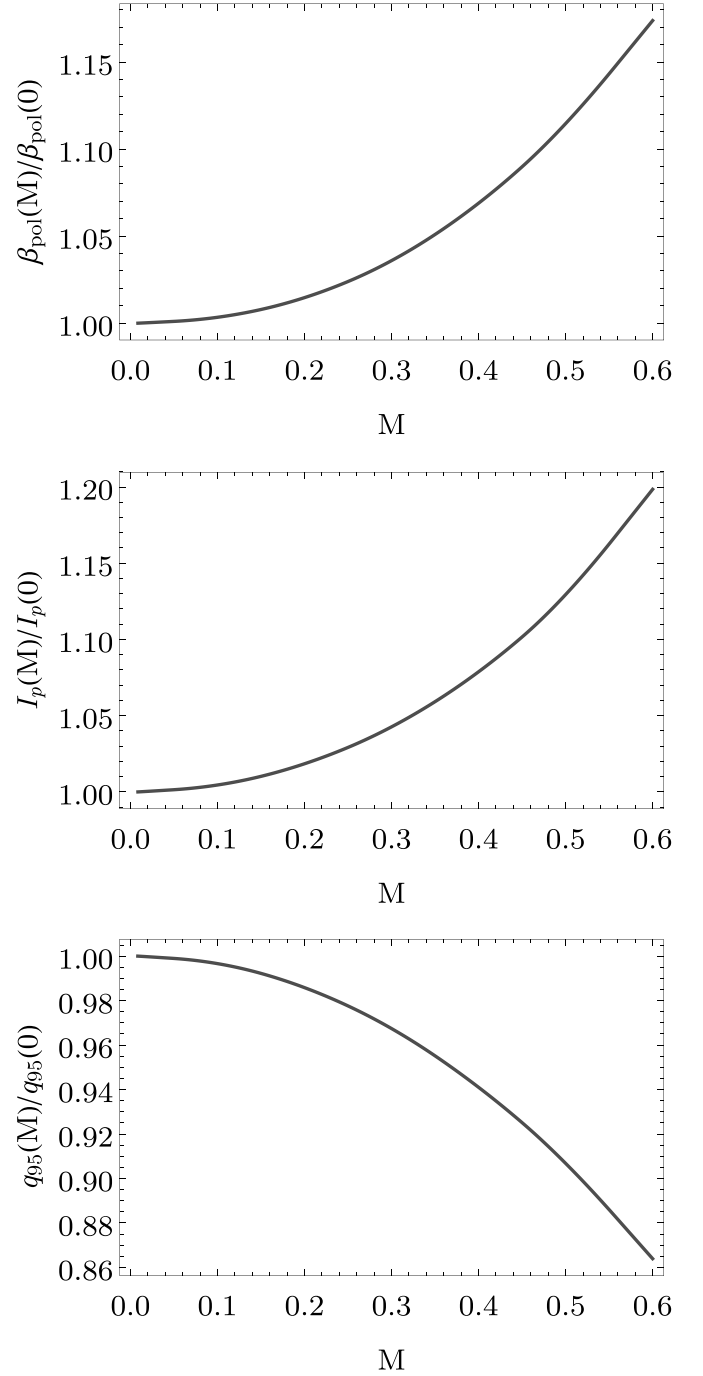


Figure 3. Variation of poloidal beta, plasma current and safety factor at 95% plasma volume, with respect to the parameter M , each normalized by their respective value in the static case. The other parameters of the configuration are kept fixed and correspond to table 1, second row.

of x -points, i.e. boundary points with vanishing magnetic field gradient, can be fixed exactly only at the cost of neglecting other constraints. This issue could be overcome by extending the expression in equation (18) up to a suitable higher power of Z , thus generating new free constants in the solution. However, we show here that the general solution of the homogeneous equation $\Delta^* \psi_H = 0$ can be written in a compact form,

without any truncation, and that it can be used to solve this kind of problem.

To find the general solution, we express ψ_H as a Fourier transform in the Z variable:

$$\psi_H(R, Z) = \int_{-\infty}^{\infty} \chi(R, k) e^{ikZ} dk. \quad (27)$$

Its reality is ensured by the condition $\chi(R, -k) = \overline{\chi(R, k)}$ (the bar indicates the complex conjugate). Plugging equation (27) into $\Delta^* \psi_H = 0$, we obtain an ordinary differential equation in the variable R for each k . By making the change of variables $x_k = |k|R$ and $\chi(R, k) = R\epsilon(x_k, k)$ (for $k \neq 0$), it is easy to verify that:

$$x_k^2 \epsilon(x_k, k)'' + x_k \epsilon(x_k, k)' - (1 + x_k^2) \epsilon(x_k, k) = 0, \quad (28)$$

where the prime denotes differentiation with respect to x_k . This is known as Bessel's modified equation with index 1, and its solution is readily available in mathematical literature:

$$\epsilon(x_k, k) = a_k I_1(x_k) + b_k K_1(x_k), \quad (29)$$

where a_k, b_k are functions of k . By substitution back into equation (27), we obtain:

$$\psi_H(R, Z) = R \int_{-\infty}^{\infty} [a_k I_1(|k|R) + b_k K_1(|k|R)] e^{ikZ} dk, \quad (30)$$

which is the general solution of the homogenous problem.

This expression can be adapted to a given scenario by imposing specific boundary conditions. In this respect, for the sake of simplicity, we represent the functions a_k, b_k as a sum of sufficiently narrow Gaussians (i.e. delta functions), centered around arbitrarily given wave vectors k_i and weighted by amplitudes \bar{a}_i, \bar{b}_i , thus obtaining:

$$\psi_H(R, Z) = R \sum_{i=1}^N [\bar{a}_i I_1(R|k_i|) + \bar{b}_i K_1(R|k_i|)] \cos(k_i Z), \quad (31)$$

where the term $\cos(k_i Z)$ is the reduction of the complex exponential to the real, up-down symmetric case. Then, a given set of points $\{r_l, z_l\}$ lying along the boundary curve of the addressed plasma configuration generates an associated set of algebraic equations of the form $\psi(r_l, z_l) = 0$, which can be solved to determine the arbitrary constants.

4.1. DTT double-null configuration

We illustrate this procedure in the practical case of the double-null plasma scenario predicted for the upcoming DTT experiment. Its main parameters are reported in table 2, and are available in [13], along with the predicted separatrix shape.

We proceed as follows: firstly, we model the desired separatrix as an analytic curve, using a piecewise rational expression (e.g. quadratic). Secondly, we extract a set of boundary points chosen at random but equally distributed around the plasma region. Thirdly, the set of wavenumbers k_i is chosen as

Table 2. Main plasma parameters of the DTT double-null scenario, taken from [13].

I_p (MA)	β_{pol}	q_{95}	R_0 (m)	a (m)	δ	κ
5.00	0.43	2.80	2.11	0.64	0.45	1.92

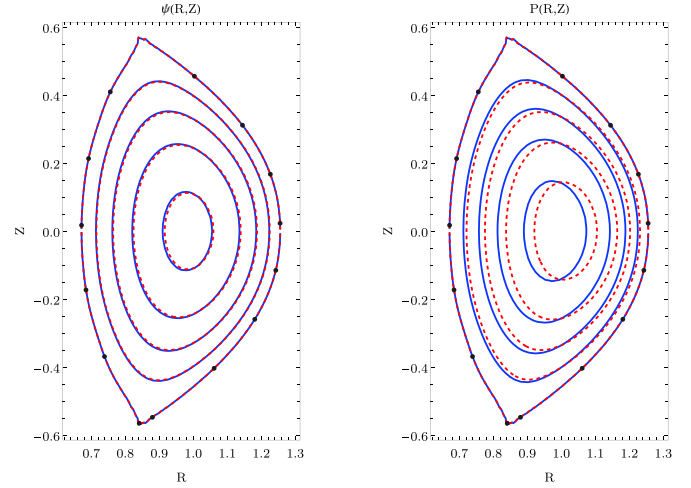


Figure 4. Contours of constant ψ (left) and P (right) in the (R, Z) plane for the DTT double-null plasma, in the static case ($M = 0$, solid blue) and rotating case ($M = 0.6$, dashed red).

an equally distributed grid of values close to the scale length of the configuration, estimated as $\pi/(a\kappa)$. The solution of the resulting set of algebraic equations gives the constants \bar{a}_i, \bar{b}_i as functions of P_1, I_1 and M . The former two are still obtained according to equations (24)–(26), and we can study the behaviour of the equilibrium for different values of M .

Figure 4 shows the fitted magnetic configuration and the curves of constant pressure, where we highlighted the correspondence between the set of boundary fitting points and the obtained separatrix. In the presence of rotation, the qualitative behaviour of the plasma is still that of an outward shift of magnetic and pressure lines, while the separatrix is kept fixed by the imposed constraints and has no major modifications.

Concerning the safety factor profile, plotted in figure 5, we predict a slight suppression in the core region, while q actually increases closer to the plasma boundary, contrary to the behaviour observed in the previous section using solution (18). However, this should not be interpreted as a fundamental difference between the two analytical approaches. In fact, even though the DTT plasma looks similar to the one shown in figure 2 (second row), their magnetic flux profiles are different, as shown in figure 6. By virtue of equations (14) and (25), also the current density profiles are different, hence we have no reason to believe their behaviour should be the same with respect to the safety factor. Concerning the equivalence of the two methods, in the same figure we also show the profile obtained from the Bessel expression (31) fitted on a set of points lying along the boundary of the polynomial solution of figure 2 (second row). This results in essentially the

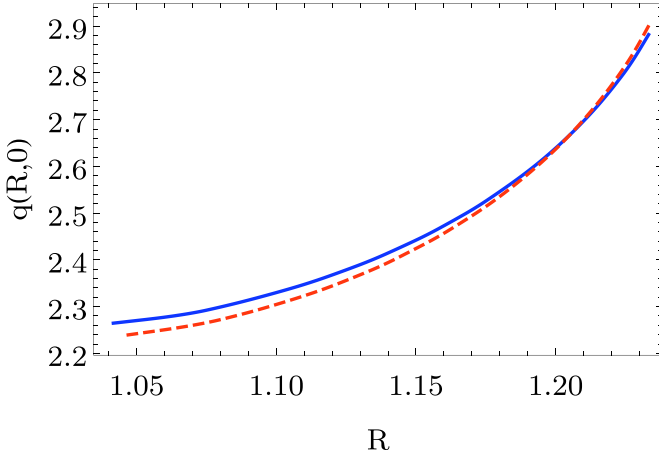


Figure 5. Safety factor profile of the DTT double-null scenario in the static case ($M = 0$, solid blue) and rotating case ($M = 0.6$, dashed red).

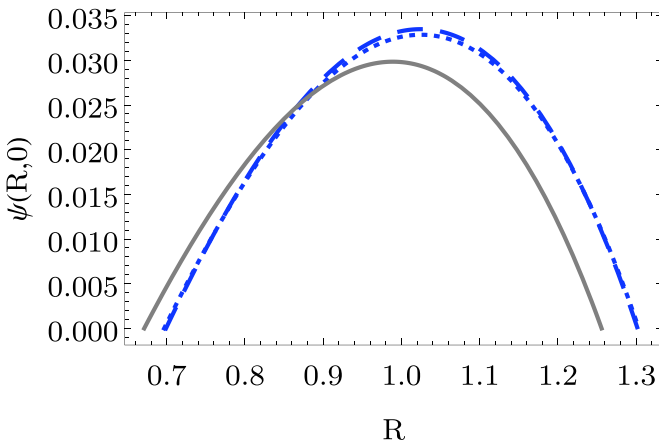


Figure 6. Equatorial profile of the magnetic flux function for the DTT case (solid grey), and the scenario of table 1, second row, fitted with solution (18) (dashed blue) and (31) (dotted blue), assuming the same values of β_{pol} and I_{pl} as in DTT.

same plasma profile, thus showing the compatibility of the two methods.

Finally, we can plot the curves of constant toroidal speed $\omega(\psi)R$ by assuming a simple form for the temperature, taken as $T(\psi) = T_{edge} + \frac{\psi}{\psi_{axis}}(T_{core} - T_{edge})$, with the temperature values according to [13]. The result is shown in figure 7, along with the morphology of pressure lines in the vicinity of the x -point. In this formalism, we do not expect serious modifications to the shape of the plasma in this region, having imposed our constraints along the separatrix itself. However, observing how the pressure (and its gradient) are suppressed in the presence of rotation might provide useful information when considering transport dynamics. For example, it is common practice to feed equilibrium data obtained from a given solver into a separate code which simulates particle transport. A scenario in which the parameter M is measured to be consistently far from 0, giving rise to noticeable modifications of the equilibrium, would need to take plasma rotation into account. Of course, the

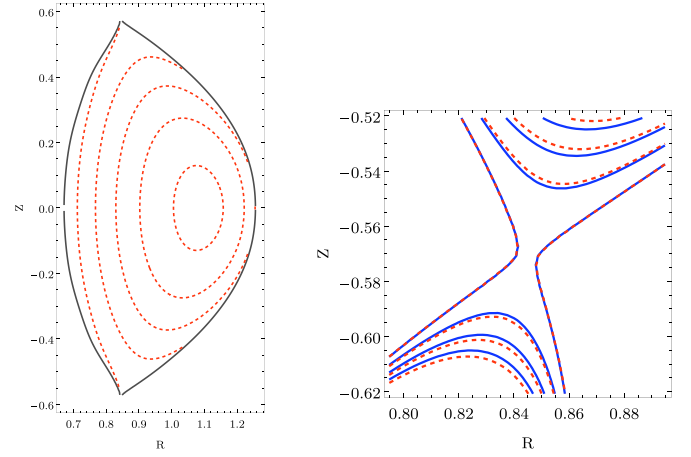


Figure 7. Contours of constant toroidal velocity ωR over the whole configuration (left), and contours of constant pressure in the vicinity of the x -point (right, $M = 0$ solid blue, $M = 0.6$ dashed red).

present analysis is aimed at providing a simple semi-analytical tool to gain insight in this direction, while accurate equilibrium solvers with plasma flow should be used for more elaborate analysis (e.g. [14]).

5. Concluding remarks

In this work, we studied the equilibrium of an axisymmetric plasma in the presence of rotation along the toroidal direction. After recalling the mathematical basic formalism, we adopted suitable assumptions on the arbitrary functions in order to obtain analytic plasma profiles, with enough freedom to represent a variety of plasma settings. In particular, the polynomial expression of section 3 requires to fix only few basic plasma parameters, such as the minor radius and the triangularity. This simplicity allows to find analytical expressions for the plasma separatrix and to deal with a variety of scenarios, e.g. double-null and negative triangularity. Then, in section 4, we presented the general solution of the considered problem, and illustrated a suitable fitting procedure when dealing with a known plasma separatrix (either analytically or numerically). As a practical implementation of this framework, we studied the double-null plasma scenario proposed for the upcoming Italian experiment DTT, estimating the impact of plasma rotation on the equilibrium properties and highlighting some points of interest. One example is the modification of the plasma pressure gradient morphology in the vicinity of the x -point, with possible effects on particle transport dynamics in that region.

Of course, the analysis performed here has the merit of simplicity due to its analytic nature, but needs to be confirmed by more detailed numerical studies when dealing with more realistic situations. Moreover, many physical constraints here neglected (e.g. the specifics of the given tokamak magnetic coils, or its current drive mechanism) would need to be taken into account. Nevertheless, the two approaches of sections 3 (reduced) and 4 (general) agree on the qualitative behaviour

of the plasma parameters as functions of the rotation velocity, hence they can both be used as quick investigative tools concerning the introduction of toroidal rotation in tokamak plasma equilibria. In experimental situations, the parameter M can be estimated from equation (14) providing direct measurements of ion rotation speed and temperature, e.g. through diagnostics like charge exchange recombination spectroscopy [21]. Depending on the value of M , quantitative estimates on the relevance of plasma rotation can be argued by the methods outlined here.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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