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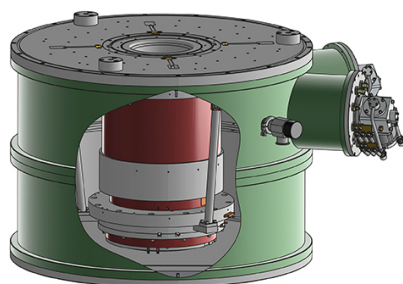


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Letter

Drift wave solitons and zonal flow: implications for staircase formation

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Abstract

The self-consistent nonlinear interaction of drift waves (DWs) and zonal flow (ZF) is investigated using nonlinear gyrokinetic theory, with both spontaneous excitation and beat-driving of ZF by DWs treated on the same footing. DW solitons are formed in nonlinear DW–ZF interactions and are confined between radially spaced micro-barriers induced by spontaneously excited ZF (SZF). The resulting radial structures in nonlinear DW–ZF interactions exhibit a similar pattern to the $\mathbf{E} \times \mathbf{B}$ ‘staircase’ observed in numerical simulations. These micro-barriers are generated by the repulsive response due to SZF, which, as a general property demonstrated in this work, also generates an attractive nonlinear potential in the DW equation. Meanwhile, the nonlinear potential due to beat-driven ZF is always attractive and, as such, always serves as a potential well to contribute to soliton formation. For SZF from initial noise, the simultaneous excitation of solitons and micro-barriers is found to be universal due to the zero-frequency nature of ZF and the spatial structure of the Reynolds stress. The present analysis thus provides a potential first-principles-based interpretation of the $\mathbf{E} \times \mathbf{B}$ staircase observed in simulations, which may contribute to the formation of micro transport barriers and enhance plasma confinement.

Keywords: drift wave, zonal flow, soliton, spontaneous excitation, beat-driven, staircase structure

(Some figures may appear in colour only in the online journal)

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Anomalous transport, generally considered to result from drift wave (DW) turbulence driven by thermal plasma nonuniformity, is able to induce significant thermal plasma transport and degradation of plasma confinement [1]. Thus, it is important to investigate its linear excitation, nonlinear saturation and propagation. Nonlinear generation of zonal flow (ZF) is considered to be an important component of self-regulation of DWs [2] and fishbone instability [3]. ZF is an $n=0$ and $m \simeq 0$ radial electric field, consisting of zero-frequency ZF [4] and a finite frequency counterpart geodesic acoustic mode (GAM) [5]. In the following context, when we refer to ZF it is the zero-frequency component of ZF. Here, n/m are toroidal and poloidal mode numbers.

There are two mechanisms for the excitation of ZF by DWs. The first is spontaneous excitation of ZF via modulational instability [6], which requires the DW intensity to exceed a threshold and also incorporates scattering of DWs into a short-wavelength stable domain, leading to self-regulation and saturation of DW turbulence. The other is thresholdless ZF beat-driven excitation [7, 8], with the growth rate of ZF being twice the instantaneous growth rate of the DWs [9, 10]. In recent works, we have demonstrated that as the amplitude of ZF reaches certain threshold, DW solitons can form in the DW–ZF system, taking into account one of the mechanisms for ZF excitation by DWs in the fully nonlinear stage [11, 12] due to the nonlinear trapping induced by nonlinearly generated ZF. The solitons are characterized by their shape and amplitude remaining unchanged during propagation. Thus they can potentially contribute to nonlocal transport via turbulence spreading from linearly unstable regions to stable regions, which is detrimental to plasma confinement [13–15]. These two mechanisms have been investigated separately in previous analytical studies, and a theoretical analysis of the DW–ZF system with both ZF generation mechanisms accounted for on the same footing was recently developed, showing a significant reduction of the threshold for modulational instability due to beat-driven ZF (BZF) [8]. Consequently, for both academic interest and its implications for confinement in fusion reactors, it is important to investigate nonlinear soliton dynamics in a fully nonlinear DW–ZF system accounting for both routes to ZF excitation.

It is also observed that a self-organizing $\mathbf{E} \times \mathbf{B}$ flow pattern is generated in numerical simulations; this is called a ‘staircase’ and changes the equilibrium temperature profile significantly [16, 17]. A staircase is a global, long-lived self-organizing phenomenon during which the smooth temperature profile evolves into a spatially quasi-regular step-like structure due to similarly distributed micro-barriers, which may improve thermal plasma confinement. More specifically, in regions between neighboring micro-barriers, avalanche-like transport induced by DWs can be observed, which flattens the pressure profile to form the ‘tread’; meanwhile, heat and particle fluxes are reduced at micro-barriers, leading to the formation of ‘steps’. These micro-barriers are believed to result from meso-scale ZF [4]. The formation of staircase structures shows that microscopic DWs can be regulated by

meso-scale ZF to significantly affect the global pressure profile. Despite its crucial role in understanding self-organizing processes in plasmas and determining overall plasma performance, the mechanism for staircase formation is still under active investigation [18, 19].

In this work, a paradigm model of ZF generation by DWs is investigated using nonlinear gyrokinetic theory, with both beat-driven and spontaneous excitation of ZF taken into account on the same footing. We have found simultaneous excitation of solitons and micro-barriers due to spontaneous excitation of ZF by DWs, resulting in confinement of DW solitons within neighboring micro-barriers for finite-amplitude DW pulses as well as for more realistic conditions with initial noise-level DWs having a finite linear growth rate. This phenomenon might serve as a potential explanation for the ‘staircase’ structure found in simulations. The width and height of micro-barriers are essentially determined by the radial mode structure of the corresponding DW envelope, and thus local micro-barriers are permeable for the larger-amplitude solitons generated elsewhere. The dual role of spontaneously excited ZF to have both attractive and repulsive nonlinearity in the DW equation is embedded in the structure of Reynolds stress and the zero-frequency nature of ZF. Meanwhile, it is not sensitive to the linear dispersion relation of DWs; this makes the present analysis general and its findings should be applicable to different types of DW.

In this work, we consider the excitation of ZF by a single- n electrostatic DW in toroidal geometry, with BZF and spontaneously excited ZF (SZF) being treated on the same footing. The two-field description of the nonlinear DW–ZF system is adopted, in which the DW is treated as a whole [11] instead of being separated into a pump wave and relatively small sidebands [6]. The single- n DW and ZF fluctuations can be expressed as

$$\begin{aligned}\delta\phi_n &= A_n(r, t) e^{in\xi - i\omega_n t} \sum_m e^{-im\theta} \Phi(nq - m) + c.c., \\ \delta\phi_Z &= A_Z(r, t) e^{ik_z r - i\omega_Z t} + c.c.,\end{aligned}$$

where A_n and ω_n are the meso-scale radial envelope and frequency of the DW, A_Z and ω_Z are the meso-scale radial structure and frequency of ZF, which vanishes for the zero-frequency component considered here [4], $\Phi(nq - m)$ is the microradial structure due to the parallel wavenumber ($k_{\parallel}$) spectrum, q is the safety factor and $c.c$ represents the complex conjugate. The subscripts ‘ n ’ and ‘ Z ’ represent the quantities associated with the DW and ZF, respectively, and $k_r \equiv -i\partial_r$ is the operator for the radial derivative. For simplicity of discussion, a large-aspect-ratio tokamak with concentric circular magnetic surfaces is considered, and the magnetic field is given by $\mathbf{B} = B_0[\mathbf{e}_{\xi}/(1 + \epsilon \cos \theta) + \epsilon \mathbf{e}_{\theta}/q]$, where $\epsilon \equiv r/R \ll 1$ is the inverse aspect ratio, R and r are the major and minor radii of the tokamak and ξ and θ are the toroidal and poloidal angles, respectively. Without loss of generality, constant temperatures are considered for different particle species, such

that plasma nonuniformity is accounted for by density nonuniformity. Particle responses to low-frequency fluctuations, such as the DW and ZF investigated here, are governed by the nonlinear gyrokinetic equation [20]

$$(\omega + iv_{\parallel}\partial_l + \omega_{Ds})\delta H_{k,s} = \frac{q_s}{T_s}(\omega - \omega_{*s})J_k\delta\phi_k F_{0s} - i\frac{c}{B_0}\Lambda_{k',k''}^k J_{k'}\delta\phi_{k'}\delta H_{k'',s}. \quad (1)$$

Here, $\omega_{*s} \equiv k_{\theta}cT_s/(eBL_N)$ is the diamagnetic frequency accounting for density nonuniformity, $L_N \equiv -N/(\partial N/\partial r)$ is the characteristic length of density nonuniformity, N is the equilibrium particle density, ∂_l represents the derivative along the equilibrium magnetic field line, $\omega_{Ds} \equiv \hat{\omega}_{ds}C$ is the magnetic drift frequency, with $\hat{\omega}_{ds} \equiv \omega_{ds}(v_{\perp}^2/2 + v_{\parallel}^2)/v_{ts}^2$, $C \equiv \cos\theta - \sin\theta k_r/k_{\theta}$, $\omega_{ds} \equiv k_{\theta}cT_s/(eBR)$ and $v_{ts} \equiv \sqrt{2T_s/m_s}$ is the thermal velocity. Furthermore, $\delta H_{k,s}$ is the nonadiabatic particle response, F_{0s} is the equilibrium distribution function, which is taken here as Maxwellian, $J_k \equiv J_k(k_{\perp}\rho_s)$ is the Bessel function of zero index describing the finite Larmor radius effect, $k_{\perp} = \sqrt{k_r^2 + k_{\theta}^2}$ is the perpendicular wavenumber, $\rho_s \equiv v_{\perp}/\omega_{cs}$ is the Larmor radius and $\omega_{cs} \equiv eB/(m_sc)$ is the cyclotron frequency. Subscripts k, k', k'' denote DW or ZF, and $s = i/e$ represents particle species. The second term on the right-hand side of equation (1) is the perpendicular nonlinear term, with the selection rules on frequency and wavenumber matching conditions for mode-mode coupling applied, where $\Lambda_{k',k''}^k \equiv \mathbf{b} \cdot (\mathbf{k}'' \times \mathbf{k}')$, $\mathbf{b} \equiv \mathbf{B}/B_0$ is the unit vector along equilibrium magnetic field lines, and other notations are standard. From that term, the excitation of ZF by a DW can be identified and found to have two routes, one is beat-driven excitation, with $k_r' = k_r''$ [12], and the other is spontaneous excitation, with $k_r' \neq k_r''$ [6]. Consequently, ZF can be written as $\delta\phi_Z \equiv \delta\phi_{Z,S} + \delta\phi_{Z,B}$, with two components representing the scalar potential of SZF and BZF, respectively. The field equations are closed by the charge quasi-neutrality condition

$$\frac{e^2N}{T_i} \left(1 + \frac{1}{\tau}\right) \delta\phi_k = \langle eJ_0\delta H_i \rangle_k - \langle e\delta H_e \rangle_k. \quad (2)$$

Here, $\tau \equiv T_e/T_i$ is the ratio of electron and ion temperature. With the particle responses derived from the nonlinear gyrokinetic equation and substituted into equation (2), the set of coupled nonlinear equations describing the nonlinear interactions between ZF and DW are given as

$$\omega_n \mathcal{E}_d \delta\phi_n = -\frac{c\tau}{B_0} k_{\theta n} \frac{\omega_{*i,n}}{\omega_n} \delta\phi_n (\partial_r \delta\phi_{Z,S} + \partial_r \delta\phi_{Z,B}), \quad (3)$$

$$\partial_l \chi_{iz} \delta\phi_{Z,S} = -\frac{c}{B_0} \alpha_i k_{\theta n} \rho_{ti}^2 \partial_r (\delta\phi_n \partial_r^2 \delta\phi_n^* - \delta\phi_n^* \partial_r^2 \delta\phi_n), \quad (4)$$

$$\chi_{iz} \delta\phi_{Z,B} = \frac{c}{B_0} k_{\theta n} \frac{\omega_{*i,n}}{\omega_n^2} |\partial_r \delta\phi_n|^2. \quad (5)$$

Here, $\mathcal{E}_d$ is the linear dispersion relation of the DW, χ_{iz} is the neoclassical inertia enhancement due to the finite drift orbit width effect, $\alpha_i \equiv 1 - \omega_{*i,n}/\omega_n$ and $\rho_{ti} \equiv v_{ti}/\omega_{ci}$. Equation (3) represents the radial envelope modulation of the DW by SZF

and BZF, while equations (4) and (5) represent spontaneous and beat-driven excitation of ZF by the DW, respectively. It is noteworthy that equations (3)–(5) were derived in [8]. For simplicity of analysis, while keeping the necessary physics elements, we adopt the linear dispersion relation of an electron DW, which is given by $\omega_n \mathcal{E}_d = \omega - \omega_{*i,n} - C_d \omega_{*i,n} k_r^2 \rho_{ti}^2 + i\mathcal{E}_l$, with C_d representing the dispersiveness of the DW [21]; $\mathcal{E}_l$ is formally the imaginary part of the linear dispersion relation, which originates from the non-adiabatic response of electrons. Meanwhile, $\chi_{iz} \approx 1.6q^2 k_{rZ}^2 \rho_{ti}^2 / \sqrt{\epsilon}$ in the long-wavelength limit of interest [22]. Substituting equation (5) into the DW equation (3), the nonlinear evolution equations for the radial envelope of the DW and SZF are given by

$$(\partial_t - \gamma_L - i\tau\Omega(r) - i\tau C_d \partial_r^2) A_n = i\beta A_n \delta E_{Z,S} + i\frac{\sqrt{\epsilon}}{1.6q^2} \beta^2 |A_n|^2 A_n, \quad (6)$$

$$\partial_t \delta E_{Z,S} = -i\frac{\sqrt{\epsilon}}{1.6q^2} \beta \alpha_i \partial_r (A_n \partial_r A_n^* - A_n^* \partial_r A_n), \quad (7)$$

where space and time are normalized by ρ_{ti} and $1/\omega_{*i,n}$ and scalar potential is normalized by e/T_i . Here, $\Omega(r) \equiv \omega_{*i,n}(r)/\omega_{*i,n}(0)$ represents the nonuniformity induced by the ion diamagnetic well, $\beta \equiv (k_{\theta n} \rho_{ti}/2) \omega_{ci}/\omega_{*i,n}$ is the nonlinear coupling coefficient, γ_L is the linear growth rate of the DW, given by the imaginary part of the linear dispersion relation, and $\delta E_{Z,S} \equiv -\partial_r \delta\phi_{Z,S}$ represents the radial electric field of SZF. Note that equation (7) is the same as equation (4) in [11] except for the factor of $\sqrt{\epsilon}/(1.6q^2)$ decrease in the nonlinear coupling coefficient, connected with inertial enhancement in SZF excitation due to neoclassical effects in toroidal plasmas. The second term on the right-hand side of equation (6) represents the nonlinear modulation due to BZF, resulting in a cubic nonlinear term [12]. In fact, the nonlinear terms in equation (6) serve as a nonlinear potential for the DW, given by $-\beta \delta E_{Z,S} - \sqrt{\epsilon} \beta^2 |A_n|^2 / (1.6q^2)$. Thus, BZF always contributes as an attractive potential well that nonlinearly traps DW envelopes. Meanwhile, the contribution of SZF depends on its sign/direction, i.e. $\delta E_{Z,S} > 0$ for a potential well (attractive nonlinearity) and $\delta E_{Z,S} < 0$ for a potential barrier (repulsive nonlinearity). Here, we note that the spatio-temporal spectral features of BZF and SZF are expected to be quite different, based on their explicit expressions, with the latter weighing more the high- k_r response of the DW. When nonlinear trapping induced by SZF and BZF balance the linear dispersiveness of the DW, DW solitons can be formed [11, 12]. The coupled equations (6) and (7) are solved numerically using the pseudo-spectral method, with the default values given by $\beta = 10\sqrt{10}$, $\alpha_i = \tau = C_d = 1$ and $\sqrt{\epsilon}/(1.6q^2) = 0.1$ for a typical tokamak parameter regime [12].

We demonstrate first that in the case with constant ω_{*i} , i.e. $\Omega(r) = 1$, model equations (6) and (7) can produce results similar to the staircase structures observed in simulations [16, 17]. Here, regularly spaced DW envelopes with random phase and noise-level SZF are adopted as initial conditions, whose spatio-temporal evolution is shown in figures 1(a) and (b). The simultaneous excitation of solitons that enhance turbulence spreading and micro-barriers localized at corresponding

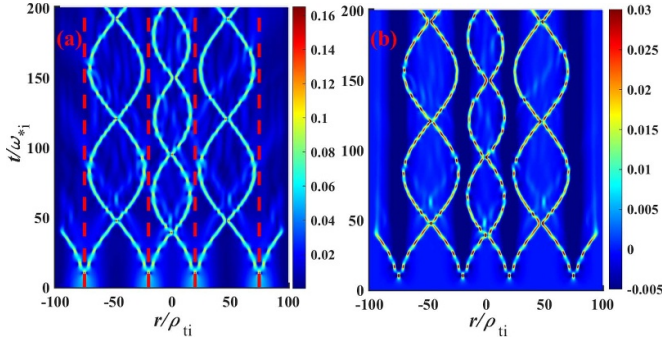


Figure 1. Spatio-temporal evolution of (a) DWs and (b) SZF with regularly spaced DW pulses with constant ω_{*i} , with the red dashed lines indicating the position of micro-barriers.

DW peaks that hinder the penetration of DW solitons can be identified. In fact, soliton formation due to ZF excitation was systematically investigated in [11, 12], which demonstrated that solitons form as the nonlinear trapping effect induced by ZF excited by DW balances the dispersiveness. However, in figure 1(b), it is found that SZF contributes not only to the nonlinear potential well with $\delta E_{Z,S} > 0$ but also to stationary potential barriers with $\delta E_{Z,S} < 0$ localized around corresponding peaks of the initial DW envelopes, which serve as micro-barriers to barricade soliton propagation and reflect solitons. Consequently, a staircase-like structure can be observed, with soliton propagation confined within radially spaced micro-barriers denoted by the red dashed lines in figure 1(a). The micro-barriers exhibit a long-lived characteristic, consistent with simulation results [16]. Note that the reflection of DW solitons presented here cannot be due to BZF, which always contributes to attractive nonlinearity as shown above. Thus, BZF contributes to soliton formation by increasing the depth of the nonlinear potential well, and eventually leads to a peaked radial mode structure. Meanwhile, it is clear that SZF can contribute to micro-barrier formation at radial locations where DW intensity peaks and the high- k_r component of the DW spectrum becomes increasingly more important, consistent with our earlier remarks.

To understand the mechanism of micro-barrier formation, a single DW envelope localized around the origin was investigated to better delineate the underlying physics. It is observed in figure 2(a) that the DW envelope splits into two counter-propagating solitons, which propagate outward without being reflected because the micro-barrier, denoted by the red dashed line in figure 2(a), is localized around the peak of the initial DW envelope, i.e. the origin in this case. This is a possible reason why the generation of a micro-barrier was not noted in previous studies on SZF [11]. The simultaneous generation of potential wells and barriers by SZF nonlinearity can be analyzed by multiplying A_n^* by equation (6), subtracting its complex conjugate and substituting into equation (7). The nonlinear evolution equation for SZF can be cast as

$$\partial_t \delta E_{Z,S} = \frac{\sqrt{\epsilon}}{1.6q^2} \frac{\beta \alpha_i}{\tau C_d} \partial_t |A_n|^2.$$

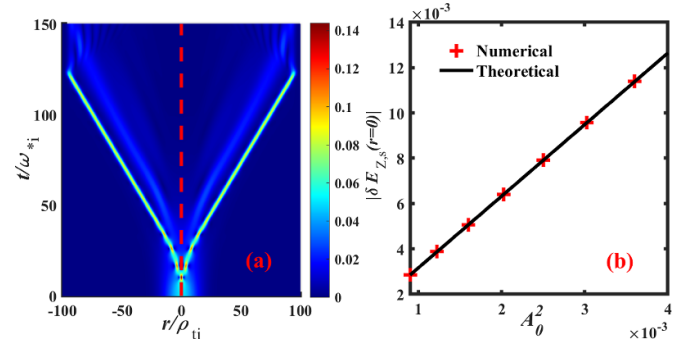


Figure 2. (a) Spatio-temporal evolution of a DW with a single initial pulse; the red dashed line indicates the position of the micro-barrier. (b) Dependence of the asymptotic value of SZF amplitude on the initial intensity of the DW; the black solid line and red crosses indicate the theoretical and numerical results, respectively.

Integrating the above equation over t and recalling that ZF is at noise level initially, the solution for SZF is given by

$$\delta E_{Z,S} = \frac{\sqrt{\epsilon}}{1.6q^2} \frac{\beta \alpha_i}{\tau C_d} \left[|A_n|^2 - |A_n|^2(t=0) \right], \quad (8)$$

where $|A_n|(t=0)$ is the initial envelope of the DW. Equation (8) demonstrates that SZF excitation is determined by the difference of the DW envelope from its initial value, which in turn is a reflection of the available free energy in the considered equilibrium. As the initial envelope splits due to the formation of DW solitons, the amplitude of the DW at the origin, where the DW envelope peaks initially, vanishes, and thus the time asymptotic value of SZF at $r=0$, i.e. the height of the micro-barriers, is given by

$$\delta E_{Z,S}(r=0) = -\frac{\sqrt{\epsilon}}{1.6q^2} \frac{\beta \alpha_i}{\tau C_d} |A_n|^2(t=0, r=0), \quad (9)$$

which is negative and proportionate to the initial amplitude of the DW, accounting for the formation of a repulsive nonlinear term at $r=0$. By measuring the time asymptotic value of SZF at the origin for different DW amplitudes from numerical studies, comparison with the analytical result given by equation (9) is shown in figure 2(b), demonstrating excellent agreement. Meanwhile, the radial structure of $\delta E_{Z,S}$ is given by $|A_n|^2$, which means that the width of the micro-barriers is comparable to that of the corresponding DW envelope.

Generally speaking, the dual nature of SZF contributing to both attractive and repulsive nonlinear responses in the DW equation is nearly universal for ZF spontaneously excited by a DW from a relatively low residual level. In fact, equation (7) is essentially a continuity equation with zero net radial flux, which after integration over the whole radial domain, and assuming $A_n \rightarrow 0$ as $r \rightarrow \infty$, yields

$$\partial_t \int_{-\infty}^{+\infty} \delta E_{Z,S} dr = 0.$$

Thus, $W \equiv \int_{-\infty}^{+\infty} \delta E_{Z,S} dr$ is a conserved quantity. Since SZF is excited from noise level initially we have $W \approx 0$, which

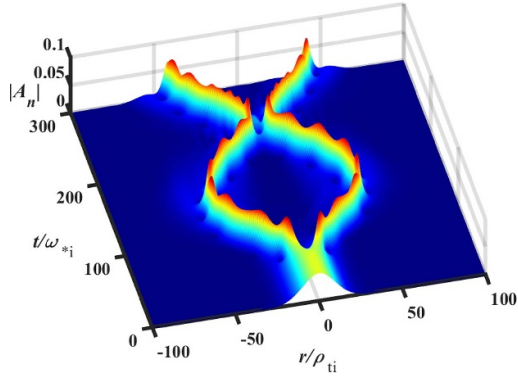


Figure 3. Spatio-temporal evolution of a single DW pulse in a nonuniform diamagnetic well.

means that the excitation of the potential well ($\delta E_{Z,S} > 0$) must be accompanied by a micro-barrier ($\delta E_{Z,S} < 0$), consistent with our analysis above. Note that the conservation of W is not affected by linear properties of the DW, i.e. $\mathcal{E}_d$, but is determined by the structure of the Reynolds stress that nonlinearly excites ZF. Consequently, the simultaneous generation of micro-barriers and solitons is general in nature, and should be observed in investigations of various types of DW and the potential implications, i.e. staircase formation.

In this work formation of micro-barriers and a staircase structure is also investigated in realistic plasmas with nonuniform $\Omega(r) = \exp(-r^2/L_p^2)$ taken into account, where L_p is the characteristic scale length of variation of $\omega_{*i}(r)$. It is observed in figure 3 that solitons are reflected back and forth by the turning points determined by the nonuniform diamagnetic well and the nonlinear micro-barrier due to SZF. In this case, a single DW pulse can itself manifest the formation of separated DW spreading regions, with the micro-barrier induced by SZF located in between them. This is different from the uniform- Ω case, where the propagation of a DW soliton is limited by the micro-barrier generated by another DW. However, we note that the first case investigated, with multiple initial DW pulses with uniform Ω , is intrinsically nonuniform, and multiple DW pulses can exist.

The above-mentioned analyses are based on given initial DW pulses to focus on the physics of micro-barrier formation. In realistic plasmas, DWs are excited from noise level as the system is linearly unstable to DWs. Meanwhile, SZF is excited as the DW amplitude is large enough for modulational instability to occur. Without loss of generality, we assume that a DW is linearly excited under spatially uniform γ_L , which is artificially turned off at t_c to impose ‘saturation’ since we have not considered the feedback to γ_L due to ZF-induced scattering. Here, t_c is the time for ‘saturation’. Similar patterns to that shown in figure 1 reappear, as shown in figure 4. Meanwhile, penetration of micro-barriers can be observed near $r = 48$, where the amplitude of solitons is sufficiently large. Recalling that the height and width of micro-barriers are determined by the corresponding DW envelopes,

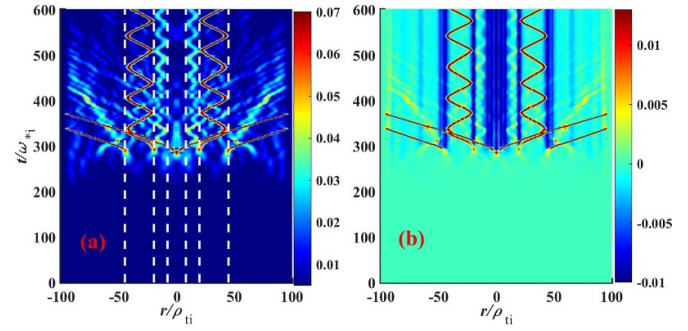


Figure 4. Spatio-temporal evolution of (a) a DW and (b) SZF with the initially noise-level DW suffering a finite linear growth rate γ_L . White lines indicate the position of SZF potential barriers. Note here that the initial noise for DW is symmetric for better visualization, $\gamma_L = 0.025$ and $t_c = 300$.

the present results demonstrate that DW solitons with relatively large amplitude, generated near the origin in figure 4, can penetrate through the micro-barriers generated by lower-amplitude DWs.

In summary, we report novel results on simultaneous excitation of solitons and micro-barriers due to spontaneous excitation of ZF by a DW using first-principles gyrokinetic theory, their potential role in explaining $\mathbf{E} \times \mathbf{B}$ staircase formation and the relevance to bulk plasma confinement enhancement in magnetically confined fusion devices. DW solitons can be confined within neighboring micro-barriers for initially finite-amplitude DW pulses, as well as for more realistic conditions with initially noise-level DWs with finite linear growth rate. The width and height of micro-barriers are essentially determined by the radial mode structure of the corresponding DW envelope, thus reflecting the available free energy in the considered nonuniform plasma equilibrium. The self-organizing state of confining DW soliton propagation within micro-barriers is beneficial for confinement improvement. Meanwhile, local micro-barriers may be permeable for larger-amplitude solitons generated elsewhere. The dual role of SZF, acting as an attractive as well as a repulsive nonlinearity in the DW equation, is embedded in the structure of Reynolds stress and the zero-frequency nature of ZF. Meanwhile, it is not sensitive to the DW linear dispersion relation, meaning that the present analysis is general and its findings should be applicable to different types of DW. GAM can also be excited by Reynolds stress of a DW as a finite-frequency component of ZF. However, it does not exhibit similar patterns of structure formation due to its finite linear group velocity [23, 24]. It is also worth noting that ZF nonlinearly generated by Alfvén eigenmodes (AEs) and/or magnetohydrodynamic (MHD) modes might affect the propagation of DWs and formation of transport barriers in a similar way to the self-generated ZF investigated in this work, which may reveal another aspect of indirect interaction of DWs and AEs/MHD modes mediated by ZF [25–27].

This work sheds light on the coexistence of solitons and micro-barriers, and suggests a possible interpretation of the

formation of self-organizing staircase structures, which may improve plasma confinement. However, the characteristics of the $\mathbf{E} \times \mathbf{B}$ staircase found in simulations, such as step size scaling to system size, permeability and near-marginal characteristics, are not addressed here. Thus, more detailed exploration via a combination of theory and simulation is necessary, and might be investigated in future publications.

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